Mini-Lecture 9.1

The Algebra of Functions; Composite Functions

Learning Objectives:

1. Add, subtract, multiply, and divide functions.
2. Construct composite functions.
3. Key vocabulary: *function composition.*

Examples:

1. For $f(x) = 2x^2 - x$ and $g(x) = 3 - 5x$, find the following.

   a) $(f+g)(x)$ b) $(g-f)(x)$ c) $(f+g)(2)$ d) $(g-f)(-2)$

   For $f(x) = 2x$, $g(x) = \sqrt{4x+5}$, and $h(x) = 2x^3 + 6x^2 + 2x$, find the following.

   e) $(f \cdot g)(x)$ f) $\left(\frac{h}{f}\right)(x)$ g) $(f \cdot g)(-1)$ h) $\left(\frac{h}{f}\right)(-3)$

2. For $f(x) = \sqrt{x-2}$ and $g(x) = 3x - 1$, find the following.

   a) $(f \circ g)(x)$ b) $(g \circ f)(x)$ c) $(f \circ g)(4)$ d) $(g \circ f)(4)$

   For $f(x) = x^2 + 3$, $g(x) = -6x$, and $h(x) = \sqrt{x-3}$, write the given $F(x)$ as a composition of *f*, *g*, or *h*.

   e) $F(x) = 36x^2 + 3$ f) $F(x) = -6x^2 - 18$ g) $F(x) = x$

   Find $f(x)$ and $g(x)$ so that the given function $h(x) = (f \circ g)(x)$.

   h) $h(x) = |x-2|$ i) $h(x) = \frac{1}{3x+5}$ j) $h(x) = \sqrt{x-2} + 4$

Teaching Notes:

- Most students do not have trouble with objectives 1 and 2.
- Some students are very confused by the concept of and mechanics of a composition function.
- Point out to students that in most situations, $(f \circ g)(x)$ and $(g \circ f)(x)$ are different.
- Refer students to the ***Algebra of Functions*** and ***Composite Functions*** charts in the text.

Answers: 1a) $2x^2-6x+3$, b) $-2x^2-4x+3$, c) -1, d) 3; e) $2x\sqrt{4x+5}$, f) x^2+3x+1, g) -2, h) 1; 2a) $\sqrt{3x-3}$, b) $3\sqrt{x-2}-1$, c) 3, d) $3\sqrt{2}-1$, e) $(f \circ g)(x)$, f) $(g \circ f)(x)$, g) $(f \circ h)(x)$, h) f(x)=\|x\|, g(x)=x-2, i) $f(x) = \frac{1}{x}$, g(x)=3x+5, j) $f(x) = \sqrt{x} + 4$, g(x)=x-2

Mini-Lecture 9.2

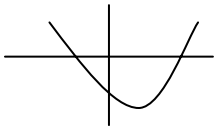
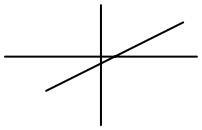
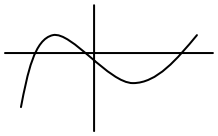
Inverse Functions

Learning Objectives:

1. Determine whether a function is a one-to-one function.
2. Use the horizontal line test to decide whether a function is a one-to-one function.
3. Find the inverse of a function.
4. Find the equation of the inverse of a function.
5. Graph functions and their inverses.
6. Determine whether two functions are inverses of each other.
7. Key vocabulary: *symmetric.*

Examples:

1. Determine whether each function is a one-to-one function.
   a) $B = \{ (6,2), (9,9), (1,4), (-1,5) \}$ b) $C = \{ (8,-1), (9,-1), (11,7), (12,2) \}$

2. Use the horizontal line test to determine whether the graph of each function is the graph of a one-to-one function.
   a) 
   b) 
   c) 

3. Find the inverse of each function.
   a) $A = \{ (1,2), (-1,3), (-3,4) \}$ b) $C = \{ (6,2), (9,9), (1,4), (-1,5) \}$

4. Find the inverse of each one-to-one function.
   a) $f(x) = 2x + 3$ b) $f(x) = \dfrac{4x-5}{3}$ c) $f(x) = \sqrt[3]{x+9}$ d) $f(x) = \dfrac{6}{7-x}$

5. Find the inverse of each function and graph the function and its inverse on the same set of axes. Graph the line $y = x$ as a dashed line.
   a) $R = \{ (-9,6), (-6,9), (3,4) \}$ b) $f(x) = 3x + 5$ c) $f(x) = x^3 - 2$

6. Determine whether functions are inverses of each other.
   a) If $f(x) = 3x + 2$, show the $f^{-1}(x) = \dfrac{x-2}{3}$
   b) f $f(x) = 2x - 7$, show the $f^{-1}(x) = \dfrac{x+7}{2}$

Teaching Notes:

- Tell students early on that f^{-1} means the inverse function of the function *f*, it does not mean $\dfrac{1}{f}$.
- Many students reduce final answers incorrectly. For example: $\dfrac{4 \pm \sqrt{5}}{8} \to \dfrac{1 \pm \sqrt{5}}{2}$.
- Most students understand the concept of an inverse better if they are told that $f^{-1}(x)$ "undoes" whatever $f(x)$ did to x, i.e. $f^{-1}(f(x)) = x$.
- Remind students to always check that their graphs of *f* and f^{-1} are symmetric about $y = x$.
- Refer students to the ***Horizontal Line Test*** and ***Finding an Equation of the Inverse of a One-to-One Function f*** charts in the text.

Answers: 1a) one-to-one, b) not one-to-one; 2a) not one-to-one, b) one-to-one, c) not one-to-one; 3a) A^{-1}*={(2,1), (3,-1), (4,-3)}, b)* C^{-1}*={(2,6), (9,9), (4,1), (5,-1)}; 4a)* $f^{-1}(x) = \dfrac{x-3}{2}$*, b)* $f^{-1}(x) = \dfrac{3x+5}{4}$*, c)* $f^{1}(x)=x^3-9$*, d)* $f^{-1}(x) = 7 - \dfrac{6}{x}$*;*
5a) R^{-1}*={(6,-9), (9,-6), (4,3)}, b)* $f^{-1}(x) = \dfrac{x-5}{3}$*, c)* $f^{-1}(x) = \sqrt[3]{x+2}$*, 6a) &6b)* $(f \circ f^{-1})(x) = x; (f \circ f^{-1})(x) = x$

Mini-Lecture 9.3

Exponential Functions

Learning Objectives:

1. Graph exponential functions.
2. Solve equations of the form $b^x = b^y$.
3. Solve problems modeled by exponential equations.

Examples:

1. Graph each exponential function.

a) $y = 3^x$ b) $y = 3^x + 1$ c) $y = 3^x - 2$

d) $y = \left(\frac{1}{2}\right)^x$ e) $y = \left(\frac{1}{2}\right)^x - 3$ f) $y = \left(\frac{1}{2}\right)^x + 2$

g) $y = -2^x$ h) $y = -\left(\frac{1}{4}\right)^x$ i) $y = 3^{x+1}$

2. Solve.

a) $3^x = 9$ b) $4^x = 1$ c) $5^x = 125$ d) $\left(\frac{1}{3}\right)^x = 27$

e) $4^x = 32$ f) $4^{x+2} = 64$ g) $\frac{1}{8} = 2^{3x}$ h) $2^{5-3x} = \frac{1}{16}$

3. Solve.

a) The rabbit population in a forest grows at the rate of 4% monthly. If there are 220 rabbits in August, find how many rabbits should be expected by next August. Use $y = 220\,(2.7)^{0.04t}$. Round to the nearest whole number.

b) Jared borrows \$3750 at a rate of 10.5% compounded monthly. Find how much Jared owes at the end of 3 years. Round to the nearest cent. Use $A = P\left(1 + \frac{r}{n}\right)^{nt}$.

Teaching Notes:

- Many students understand the graphs better if the first few are done by plotting points instead of by using shifting ideas.
- Some students find the problems in objective 2 confusing at first and need to be given a step-by-step process for solving them.

Answers: (graph answers at end of mini-lectures) 2a) 2, b) 0, c) 3, d) -3, e) $\frac{5}{2}$, f) 1, g) -1, h) 3; 3a) 354 rabbits, b) \$5,131.44

Mini-Lecture 9.4

Logarithmic Functions

Learning Objectives:

1. Write exponential equations with logarithmic notation and write logarithmic equations with exponential notation.
2. Solve logarithmic equations by using exponential notation.
3. Identify and graph logarithmic functions.

Examples:

1. Write each as an exponential equation.

   a) $\log_7 49 = 2$ b) $\log_2 16 = 4$ c) $\log_5 \frac{1}{125} = -3$ d) $\log_3 \frac{1}{9} = -2$

   e) $\log_e x = 5$ f) $\log_e \frac{1}{e} = -1$ g) $\log_{11} \sqrt{11} = \frac{1}{2}$ h) $\log_{0.5} 0.125 = 3$

   Write each as a logarithmic equation.

   i) $5^2 = 25$ j) $2^5 = 32$ k) $10^{-2} = 0.01$ l) $10^{\frac{1}{3}} = \sqrt[3]{10}$

2. Solve.

   a) $\log_2 16 = x$ b) $\log_x 64 = 3$ c) $\log_2 \frac{1}{32} = x$ d) $\log_{25} x = \frac{1}{2}$

   e) $\log_{\frac{3}{4}} x = 2$ f) $\log_x 81 = 4$ g) $\log_7 7^{-2} = x$ h) $9^{\log_9 8} = x$

3. Graph each logarithmic function.

   a) $y = \log_2 x$ b) $f(x) = \log_{\frac{1}{2}} x$ c) $f(x) = \log_{10} x$

Teaching Notes:

- Many students have trouble understanding the concept of a logarithm.
- Tell students early on that a logarithm is an exponent.
- Remind students frequently that the domain of $y = \log_b x$ is $x > 0$.
- Refer students to the ***Logarithmic Definition*** chart in the text.

Answers: (graph answers at end of mini-lectures) 1a) $7^2=49$, b) $2^4=16$, c) $5^{-3} = \frac{1}{125}$, d) $3^{-2} = \frac{1}{9}$, e) $e^5=x$, f) $e^{-1} = \frac{1}{e}$, g) $11^{\frac{1}{2}} = \sqrt{11}$, h) $0.5^3=0.125$, i) $\log_5 25 = 2$, j) $\log_2 32 = 5$, k) $\log_{10} 0.01 = -2$, l) $\log_{10} \sqrt[3]{10} = \frac{1}{3}$; 2a) 4, b) 4, c) -5, d) 5, e) $\frac{9}{16}$, f) 3, g) -2, h) 8

Mini-Lecture 9.5

Properties of Logarithms

Learning Objectives:

1. Use the product property of logarithms.
2. Use the quotient property of logarithms.
3. Use the power property of logarithms.
4. Use the properties of logarithms together.

Examples:

1. Write each sum as a single logarithm. Assume that variables represent positive numbers.

   a) $\log_3 5 + \log_3 8$ b) $\log_4 y^3 + \log_4 (y-9)$ c) $\log_2 6 + \log_2 (x+1) + \log_2 4$

2. Write each difference as a single logarithm. Assume that variables represent positive numbers.

   a) $\log_3 13 - \log_3 2$ b) $\log_5 x - \log_5 (y+1)$ c) $\log_7 (x^2+2) - \log_7 (x^2+5)$

3. Use the power property to rewrite each expression.

   a) $\log_2 x^3$ b) $\log_9 5^{-3}$ c) $\log_4 \sqrt{x}$ d) $\log_5 \sqrt[3]{y}$

4. Write each as a single logarithm. Assume that variables represent positive numbers.

   a) $\log_5 3 + \log_5 x^2$ b) $4\log_6 x + 5\log_6 y$

   c) $\log_4 14 + \log_4 2 - \log_4 7$ d) $2\log_3 x + \frac{1}{3}\log_3 x - 2\log_3 (x+1)$

   Write each expression as a sum or difference of logarithms. Assume that variables represent positive numbers.

   e) $\log_6 \frac{4x}{3}$ f) $\log_b \sqrt{6x}$ g) $\log_5 x^4(x+2)$ h) $\log_5 \frac{(x+3)^2}{x}$

   If $\log_b 3 \approx 0.5$ and $\log_b 5 \approx 0.7$, evaluate each expression.

   i) $\log_b \left(\frac{3}{5}\right)$ j) $\log_b 9$ k) $\log_b \sqrt[3]{5}$

Teaching Notes:

- Most students do not have trouble applying the properties of logarithms separately.
- Some students have trouble with objective 4, where all of the properties are combined and need to see many examples.
- Encourage students to write the three properties of logarithms on an index card for easy reference.
- Refer students to the ***Product/ Quotient/ Power Property of Logarithms*** chart in the text.

Answers: 1a) $\log_3 40$, *b)* $\log_4(y^4 - 9y^3)$, *c)* $\log_2(24x+24)$; *2a)* $\log_3\left(\frac{13}{2}\right)$, *b)* $\log_5\left(\frac{x}{y+1}\right)$, *c)* $\log_7\left(\frac{x^2+2}{x^2+5}\right)$; *3a)* $3\log_2 x$, *b)* $-3\log_9 5$, *c)* $\frac{1}{2}\log_4 x$, *d)* $\frac{1}{3}\log_5 y$; *4a)* $\log_5 3x^2$, *b)* $\log_6 x^4y^5$, *c)* $\log_4 4$, *d)* $\log_3\left(\frac{x^{\frac{7}{3}}}{(x+1)^2}\right)$, *e)* $\log_6 4 + \log_6 x - \log_6 3$, *f)* $\frac{1}{2}(\log_b 6x + \log_b x)$, *g)* $4\log_5 x + \log_5(x+2)$, *h)* $2\log_5(x+3) - \log_5 x$, *i) -0.2, j) 1, k) 0.23*

Mini-Lecture 9.6

Common Logarithms, Natural Logarithms, and Change of Base

Learning Objectives:

1. Identify common logarithm and approximate them by calculator.
2. Evaluate common logarithms of powers of 10.
3. Identify natural logarithms and approximate them by calculator.
4. Evaluate natural logarithms of powers of e.
5. Use the change of base formula.
6. Key vocabulary: *common logarithm, natural logarithm.*

Examples:

1. Use a calculator to approximate each logarithm to four decimal places.

   a) $\log 10$ b) $\log 23.1$ c) $\log 45,600$ d) $\log 0.369$

2. Find the exact value of each logarithm.

   a) $\log 1000$ b) $\log \frac{1}{100}$ c) $\log 0.001$ d) $\log \sqrt[3]{10}$

3. Identify natural logarithms.

   a) $\ln e$ b) $\ln 9.82$ c) $\ln 132,000$ d) $\ln 0.015$

4. a) $\ln e^3$ b) $\ln \sqrt[6]{e}$ c) $\ln e^{2.1}$ d) $\ln 1$

5. Approximate each logarithm to four decimal places.

   a) $\log_3 6$ b) $\log_5 9$ c) $\log_6 \frac{1}{3}$ d) $\log_{\frac{1}{2}} 3$

Teaching Notes:

- Some students need help with calculator strokes.
- Most students find the change of base formula very non-intuitive and need to try many examples to become comfortable with it.
- Many students wonder where the change of base formula comes from. Tell them they will be able to derive it in the next textbook section.
- Most students understand objective 4 after a few examples.

Answers: 1a) 1, b) 1.3636, c) 4.6590, d) -0.4330, 2a) 3, b) -2, c) -3, d) $\frac{1}{3}$*, 3a) 1, b) 2.2844, c) 11.7906, d) -4.1997;4a) 3, b)* $\frac{1}{6}$*, c) 2.1, d) 0; 5a) 1.6309, b) 1.3652, c) -0.6131, d) -1.5850;*

Mini-Lecture 9.7

Exponential and Logarithmic Equations and Applications

Learning Objectives:

1. Solve exponential equations.
2. Solve logarithmic equations.
3. Solve problems that can be modeled by exponential and logarithmic equations.

Examples:

1. Solve each equation. Give an exact solution and a four decimal place approximation.

   a) $3^x = 8$ b) $4^x = 5$ c) $5^{2x} = 6.3$

   d) $e^{2x} = 10$ e) $3^{x+8} = 7$ f) $6^{4x-5} = 18$

2. Solve each equation.

   a) $\log_2 (x+2) = 4$ b) $\log_3 7 + \log_3 x = 1$ c) $\log_4 12 - \log_4 x = 3$

   d) $\log_4 (x^2 - 3x) = 1$ e) $\log_2 x + \log_2 (x+8) = 4$ f) $\log_3 (x+2) - \log_3 x = 2$

3. Solve.

   a. The size of the deer population at a national park increases at the rate of 3.3% per year. If the size of the current population is 124, find how many deer there should be in 6 years. Use $A = A_0 e^{0.033t}$ to solve and round to the nearest whole number.

   b. Find out how long it takes a $3500 investment to earn $400 in interest if it is invested at 8% compounded semi-annually. Round to the nearest tenth of a year. Use the formula $A = P\left(1 + \frac{r}{n}\right)^{nt}$.

Teaching Notes:

- Remind students that it is not possible to take the logarithm of a negative number. Therefore some of the solutions in objective 2 must be discarded.
- Encourage students to always check their solutions in the original equation.
- Many students appreciate seeing a derivation of the change of base formula at this point:

$$\log_b x = y \;\rightarrow\; b^y = x \;\rightarrow\; \log_a b^y = \log_a x \;\rightarrow\; y \log_a b = \log_a x \;\rightarrow\; y = \frac{\log_a x}{\log_a b}$$

Answers: 1a) $\left\{\frac{\log 8}{\log 3} \approx 1.8928\right\}$, *b)* $\left\{\frac{\log 5}{\log 4} \approx 1.1610\right\}$, *c)* $\left\{\frac{\log 6.3}{2\log 5} \approx 0.5718\right\}$, *d)* $\left\{\frac{1}{2\log e} \approx 1.1513\right\}$ or $\left\{\frac{\ln 10}{2} \approx 1.1513\right\}$, *e)* $\left\{\frac{\log 7}{\log 3} - 8 \approx -6.2288\right\}$, *f)* $\left\{\frac{1}{4}\left(\frac{\log 18}{\log 6} + 5\right) \approx 1.6533\right\}$; *2a) {14}, b)* $\left\{\frac{3}{7} \approx 0.4286\right\}$, *c) {0.1875}, d) {4,-1},* *e)* $\{-4 + 4\sqrt{2} \approx 1.6569\}$, *f) {0.25}; 3a) 151, b) 1.4 yrs*

Additional Exercises 9.1
Form I

Name _____

Date _____

Find the functions $f(x) = 3x + 1$ and $g(x) = x^2 + 2x$, find:

1. $(f + g)(x)$ **1.** _____

2. $(f - g)(x)$ **2.** _____

3. $(f \cdot g)(x)$ **3.** _____

4. $\left(\frac{f}{g}\right)(x)$ **4.** _____

5. $(f \circ g)(x)$ **5.** _____

6. $(g \circ f)(x)$ **6.** _____

7. $(g \circ f)(-2)$ **7.** _____

Find the functions $f(x) = x + 9$ and $g(x) = 3x^2 + 7x$, find:

8. $(f + g)(x)$ **8.** _____

9. $(f - g)(x)$ **9.** _____

10. $(f \cdot g)(x)$ **10.** _____

11. $\left(\frac{f}{g}\right)(x)$ **11.** _____

12. $(f \circ g)(x)$ **12.** _____

13. $(g \circ f)(x)$ **13.** _____

14. $(g \circ f)(-2)$ **14.** _____

Additional Exercises 9.1
Form II

Name _____

Date _____

Find the functions $f(x) = x + 8$ and $g(x) = x^3 - 1$, find:

1. $(g + f)(x)$ **1.** _____

2. $(g - f)(x)$ **2.** _____

3. $(f \cdot g)(x)$ **3.** _____

4. $\left(\frac{g}{f}\right)(x)$ **4.** _____

5. $(f \circ g)(x)$ **5.** _____

6. $(g \circ f)(x)$ **6.** _____

7. $(g \circ f)(5)$ **7.** _____

Find the functions $f(x) = x + 12$ and $g(x) = 2\sqrt{x}$, find:

8. $(g + f)(x)$ **8.** _____

9. $(g - f)(x)$ **9.** _____

10. $(f \cdot g)(x)$ **10.** _____

11. $\left(\frac{g}{f}\right)(x)$ **11.** _____

12. $(f \circ g)(x)$ **12.** _____

13. $(g \circ f)(x)$ **13.** _____

14. $(g \circ f)(4)$ **14.** _____

Additional Exercises 9.1
Form III

Name _____

Date _____

Find the functions $f(x) = 7 - 6x$ and $g(x) = 2x^2 - 3x + 1$, find:

1. $(f + g)(x)$ **1.** _____

2. $(f - g)(x)$ **2.** _____

3. $(g - f)(x)$ **3.** _____

4. $\left(\frac{g}{f}\right)(x)$ **4.** _____

5. $(f \circ g)(x)$ **5.** _____

6. $(g \circ f)(x)$ **6.** _____

7. $(f - g)(2)$ **7.** _____

Find the functions $f(x) = -8x + 7$ and $g(x) = x^2 + x - 5$, find:

8. $(f + g)(x)$ **8.** _____

9. $(f - g)(x)$ **9.** _____

10. $(g - f)(x)$ **10.** _____

11. $\left(\frac{g}{f}\right)(x)$ **11.** _____

12. $(f \circ g)(x)$ **12.** _____

13. $(g \circ f)(x)$ **13.** _____

14. $(g \circ f)(3)$ **14.** _____

Additional Exercises 9.2
Form I

Name _____

Date _____

Determine whether each function is a one-to-one function. If it is one-to-one, list the inverse function.

1. $f = \{(0, 1), (9, 2), (8, 7), (1, 6)\}$ **1.** _____

2. $g = \{(3, 2), (1, 7), (10, -2), (2, 3)\}$ **2.** _____

3. $h = \{(7, 5), (3, 5), (-5, 2), (-1, 5)\}$ **3.** _____

4. $r = \{(0, -2), (-1, 7), (12, -2), (1, 0)\}$ **4.** _____

Each of the following functions is one-to-one. Find the inverse of each function.

5. $f(x) = 2x + 3$ **5.** _____

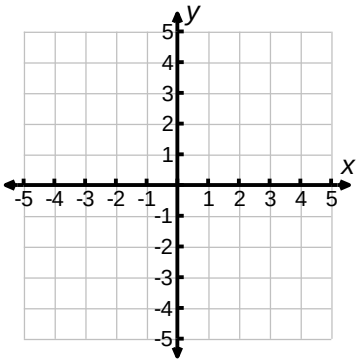
6. $f(x) = 1 - 3x$ **6.** _____

7. $h(x) = x^3 + 1$ **7.** _____

8. $h(x) = \dfrac{1}{x}$ **8.** _____

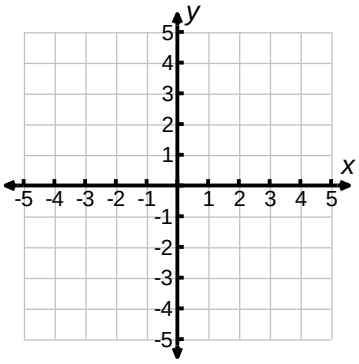
9. Graph $f(x) = 5x - 1$ and its inverse on the same set of axes.

9.



10. Graph $g(x) = -2x - 3$ and its inverse on the same set of axes.

10.



Additional Exercises 9.2
Form II

Name _____

Date _____

Determine whether each function is a one-to-one function. If it is one-to-one, list the inverse function.

1. $f = \{(5, -8), (-1, -2), (-4, 1), (2, -5)\}$ **1.** _____

2. $g = \{(2, 19), (1, 7), (-1, 7), (0, 3)\}$ **2.** _____

3. $h = \{(6, 4), (3, 2), (-3, -2), (0, 0)\}$ **3.** _____

4. $r = \{(-1, -2), (-5, -4), (3, -2), (7, -4)\}$ **4.** _____

Each of the following functions is one-to-one. Find the inverse of each function.

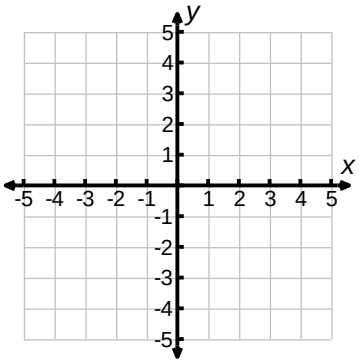
5. $f(x) = 2x - 6$ **5.** _____

6. $f(x) = 3x + 2$ **6.** _____

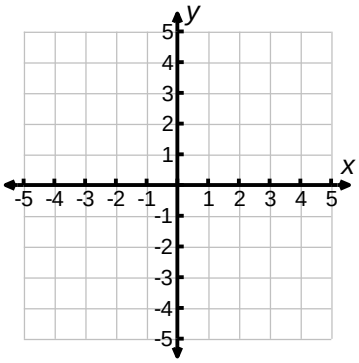
7. $h(x) = 2x^3 - 3$ **7.** _____

8. $h(x) = \sqrt[3]{x} + 1$ **8.** _____

9. Graph $f(x) = 7x - 3$ and its inverse on the same set of axes. **9.**



10. Graph $g(x) = -3x + 4$ and its inverse on the same set of axes. **10.**



Additional Exercises 9.2
Form III

Name _____

Date _____

Determine whether each function is a one-to-one function. If it is one-to-one, list the inverse function.

1. $f = \{(7, 6), (3, 1), (-7, 2), (0, 1)\}$ **1.** _____

2. $g = \{(14, -5), (7, -1), (6, 0), (5, 4)\}$ **2.** _____

3. $h = \{(3, -2), (9, 14), (0, 2), (3, 6)\}$ **3.** _____

4. $r = \{(0, -1), (2, 0), (-2, -8), (4, 1)\}$ **4.** _____

Each of the following functions is one-to-one. Find the inverse of each function.

5. $f(x) = \sqrt[3]{x+4}$ **5.** _____

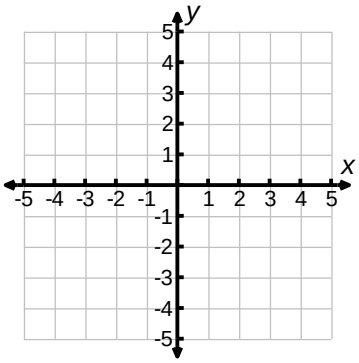
6. $f(x) = 3x^3 - 6$ **6.** _____

7. $g(x) = \dfrac{1}{3x+4}$ **7.** _____

8. $g(x) = \dfrac{x-5}{2x+5}$ **8.** _____

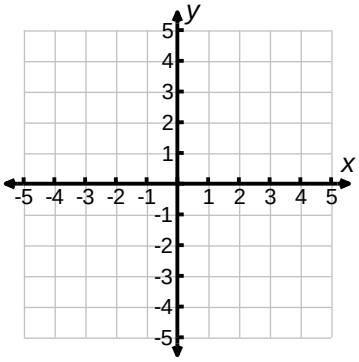
9. Graph $f(x) = \dfrac{4}{2x+1}$ and its inverse on the same set of axes.

9.



10. Graph $g(x) = x^3 - 2$ and its inverse on the same set of axes.

10.



Additional Exercises 9.3
Form I

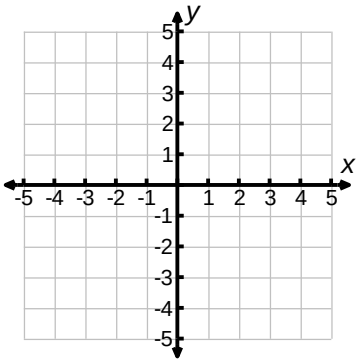
Name _____

Date _____

Graph the exponential function.

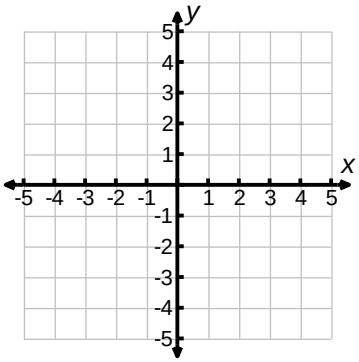
1. $y = 2^x$

1.



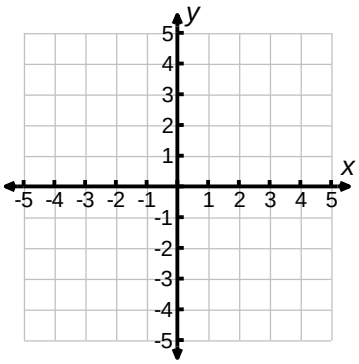
2. $y = -2^x$

2.



3. $y = \left(\frac{1}{2}\right)^x$

3.



Solve.

4. $3^x = 9$

4. _____

5. $2^x = 16$

5. _____

6. $3^x = \frac{1}{3}$

6. _____

Additional Exercises 9.3 (*cont.*)

Name _____

Solve.

7. $4^{x+1} = 16$ **7.** _____

8. $2^{-2x} = \frac{1}{16}$ **8.** _____

9. $4^{2x} = 4^{x+8}$ **9.** _____

10. $2^{x+5} = 8^{x+9}$ **10.** _____

11. $27^{x-4} = 3^{x}$ **11.** _____

12. $2^{x^2} = 4^{2x}$ **12.** _____

13. The exponential function $f(x) = 5000(2)^{0.04x}$ is a model for the population of a small community x years after 1990. Predict the population for 2015. Round to the nearest whole number. **13.** _____

14. Use $A = P\left(1+\frac{r}{n}\right)^{nt}$ to find the amount accrued if \$10,000 is invested for one year at 5% compounded quarterly. Round to the nearest cent. **14.** _____

15. A substance undergoes radioactive decay at a rate of 1% per day. For a mass of 1 kilogram of the substance, the formula $y = (2.7)^{-0.01t}$ indicates how many kilograms of the undecayed radioactive substance will remain after t years. How many kilograms of radioactive material remain after 20 days? Round to the nearest thousandths of a kilogram. **15.** _____

Additional Exercises 9.3
Form II

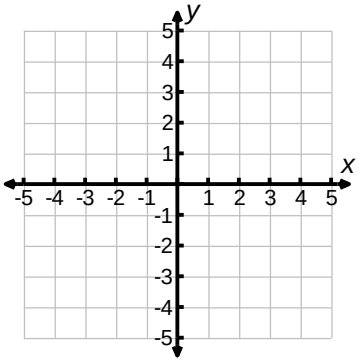
Name _____

Date _____

Graph the exponential function.

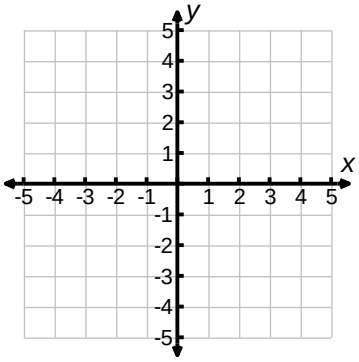
1. $y = 4^x$

1.



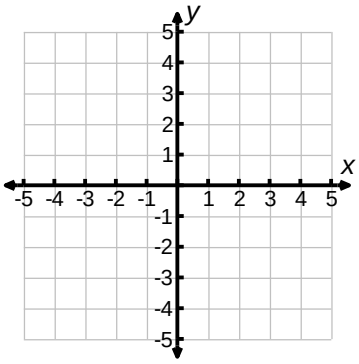
2. $y = 2 + 3^x$

2.



3. $y = -\left(\frac{1}{5}\right)^x$

3.



Solve.

4. $9^{x+1} = 81$

4. _____

5. $\left(\frac{3}{4}\right)^x = \frac{9}{16}$

5. _____

6. $3^{-x} = \frac{1}{27}$

6. _____

Additional Exercises 9.3 (*cont.*)

Name _____

Solve.

7. $16^x = 64$ **7.** _____

8. $5^{3x+1} = 25^{x-3}$ **8.** _____

9. $\left(\frac{4}{25}\right)^{2x+3} = \left(\frac{2}{5}\right)^{3x-4}$ **9.** _____

10. $6^{x-4} = 36^{2x}$ **10.** _____

11. $3^{x^2} = 9^{2x}$ **11.** _____

12. $6^{x^2} = 36^{4x}$ **12.** _____

13. The exponential function $f(x) = 25{,}000(2)^{0.05x}$ is a model for the population of a community x years after 1990. Predict the population for 2030. Round to the nearest whole number. **13.** _____

14. Use $A = P\left(1+\frac{r}{n}\right)^{nt}$ to find the amount accrued if \$15,000 is invested for one year at 4% compounded quarterly. Round to the nearest cent. **14.** _____

15. A substance undergoes radioactive decay at a rate of 0.7% per day. For a mass of 1 kilogram of the substance, the formula $y = (2.7)^{-0.007t}$ indicates how many kilograms of the undecayed radioactive substance will remain after t years. How many kilograms of radioactive material remain after 40 days? Round to the nearest thousandths of a kilogram. **15.** _____

Additional Exercises 9.3
Form III

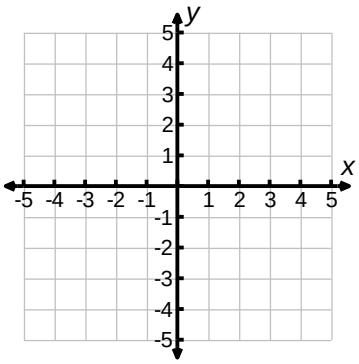
Name _____

Date _____

Graph the exponential function.

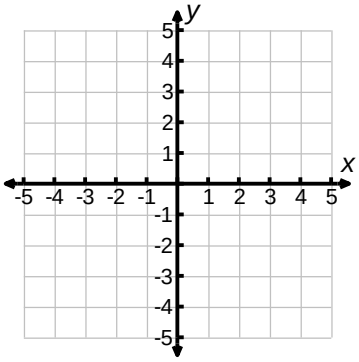
1. $y = -3^x$

1.



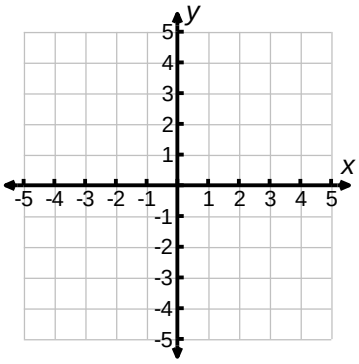
2. $y = 5^x - 4$

2.



3. $y = \left(\frac{3}{4}\right)^x + 1$

3.



Solve.

4. $64^x = \frac{1}{8}$

4. _____

5. $32^x = 16$

5. _____

6. $11^{x+1} = 121$

6. _____

Additional Exercises 9.3 (*cont.*)

Name _____

Solve.

7. $7^x = \frac{1}{343}$ **7.** _____

8. $3^{x+1} = 9^{x+5}$ **8.** _____

9. $625^{x-4} = 5^{2x}$ **9.** _____

10. $125^{x^2} = 5^{2x}$ **10.** _____

11. $5^{x^2-8} = 25^x$ **11.** _____

12. $3^{x^2-12} = 9^{2x}$ **12.** _____

13. The exponential function $f(x) = 750{,}000(2)^{0.06x}$ is a model for the population of a city x years after 1990. Predict the population for 2025. Round to the nearest whole number. **13.** _____

14. Use $A = P\left(1+\frac{r}{n}\right)^{nt}$ to find the amount accrued if \$25,000 is invested for one year at 5% compounded monthly. Round to the nearest cent. **14.** _____

15. A substance undergoes radioactive decay at a rate of 0.3% per year. For a mass of 1 kilogram of the substance, the formula $y = (2.7)^{-0.003t}$ indicates how many kilograms of the undecayed radioactive substance will remain after t years. How many kilograms of radioactive material remain after 1000 years? Round to the nearest thousandths of a kilogram. **15.** _____

Additional Exercises 9.4
Form I

Name _____

Date _____

Find the value of each logarithmic expression.

1. $\log_3 27$ **1.** _____

2. $\log_2 \frac{1}{16}$ **2.** _____

Write each as an exponential equation.

3. $\log_2 16 = 4$ **3.** _____

4. $\log_3 81 = 4$ **4.** _____

Write each as a logarithmic equation.

5. $3^2 = 9$ **5.** _____

6. $10^3 = 1000$ **6.** _____

Solve.

7. $\log_3 1 = x$ **7.** _____

8. $\log_{10} x = 0$ **8.** _____

9. $\log_2 x = 7$ **9.** _____

10. $\log_2 x = \frac{1}{2}$ **10.** _____

11. $\log_3 x = \frac{1}{2}$ **11.** _____

12. $\log_x 4 = \frac{1}{2}$ **12.** _____

13. $\log_3 (x-1) = 2$ **13.** _____

14. $\log_x 25 = 2$ **14.** _____

15. $\log_x \frac{1}{9} = \frac{1}{2}$ **15.** _____

Additional Exercises 9.4
Form II

Name _____

Date _____

Find the value of each logarithmic expression.

1. $\log_3 81$ **1.** _____

2. $\log_4 64$ **2.** _____

Write each as an exponential equation.

3. $\log_4 16 = 2$ **3.** _____

4. $\log_6 \frac{1}{36} = -2$ **4.** _____

Write each as a logarithmic equation.

5. $10^4 = 10{,}000$ **5.** _____

6. $\left(\sqrt[4]{16}\right)^3 = 8$ **6.** _____

Solve.

7. $\log_7 x = 0$ **7.** _____

8. $7^{\log_7 8} = x$ **8.** _____

9. $\log_2 x = 10$ **9.** _____

10. $\log_8 x = \frac{2}{3}$ **10.** _____

11. $\log_{\frac{2}{3}} x = 4$ **11.** _____

12. $\log_x 36 = \frac{1}{2}$ **12.** _____

13. $\log_4 (x-2) = 2$ **13.** _____

14. $\log_x 36 = 2$ **14.** _____

15. $\log_x \frac{1}{5} = \frac{1}{3}$ **15.** _____

Additional Exercises 9.4
Form III

Name _____

Date _____

Find the value of each logarithmic expression.

1. $\log_5 25$ **1.** _____

2. $\log_4 \frac{1}{64}$ **2.** _____

Write each as an exponential equation.

3. $\log_2 64 = 6$ **3.** _____

4. $\log_8 \frac{1}{2} = -\frac{1}{3}$ **4.** _____

Write each as a logarithmic equation.

5. $e^{-2} = x$ **5.** _____

6. $\left(\sqrt[5]{32}\right)^2 = 4$ **6.** _____

Solve.

7. $\log_8 x = 0$ **7.** _____

8. $\log_3 x = 6$ **8.** _____

9. $\log_{\frac{1}{4}} x = 3$ **9.** _____

10. $\log_x 5 = \frac{1}{4}$ **10.** _____

11. $\log_5 (2x - 3) = 4$ **11.** _____

12. $\log_x \frac{1}{7} = \frac{1}{5}$ **12.** _____

13. $\log_{125} x = -\frac{2}{3}$ **13.** _____

14. $\log_{\frac{27}{64}} x = 3$ **14.** _____

15. $\log_x \frac{81}{2401} = 4$ **15.** _____

Additional Exercises 9.5
Form I

Name _____

Date _____

Write each as a single logarithm. Assume that variables represent positive numbers.

1. $\log_2 5 + \log_2 3$ **1.** _____

2. $\log_2 x - \log_2 3$ **2.** _____

3. $\log_5 12 - \log_5 x$ **3.** _____

4. $\log_9 3 + \log_9 y$ **4.** _____

5. $5\log_3 x + 6\log_3 y$ **5.** _____

6. $\log_3 x - \log_3 (x-1)$ **6.** _____

7. $3\log_2 x + \frac{1}{2}\log_2 (x+3)$ **7.** _____

Write each expression as a sum or difference of multiples of logarithms.

8. $\log_3 \frac{2y}{7}$ **8.** _____

9. $\log_4 \frac{\sqrt{x}}{16}$ **9.** _____

10. $\log_4 x^2 y^3$ **10.** _____

11. $\log_5 \sqrt[4]{x}$ **11.** _____

12. $\log_4 \frac{\sqrt[3]{x}}{y^2}$ **12.** _____

13. $\log_3 x(x-1)$ **13.** _____

14. $\log_b x^3 \sqrt[5]{y}$ **14.** _____

Additional Exercises 9.5
Form II

Name _____

Date _____

Write each as a single logarithm. Assume that variables represent positive numbers.

1. $\log_3 4 + \log_3 2$ **1.** _____

2. $\log_7 14 - \log_7 x$ **2.** _____

3. $\log_9 6 - 2\log_9 y$ **3.** _____

4. $\log_8 x + \log_8 (x+4)$ **4.** _____

5. $\log_2 x + \log_2 y + \log_2 z$ **5.** _____

6. $\log_6 3x - \log_6 (2x+4)$ **6.** _____

7. $\frac{1}{2}\log_5 x + 3\log_5 (2x-1)$ **7.** _____

Write each expression as a sum or difference of multiples of logarithms.

8. $\log_2 \frac{6}{7z}$ **8.** _____

9. $\log_2 \sqrt[3]{\frac{x^4}{y^5}}$ **9.** _____

10. $\log_5 \frac{\sqrt[3]{x}}{y^2}$ **10.** _____

11. $\log_6 3x(2x-6)$ **11.** _____

12. $\log_6 \frac{5x^2+6x}{y^3}$ **12.** _____

13. $\log_{10} \frac{y}{\sqrt{2x+3}}$ **13.** _____

14. $\log_b x\sqrt{y}\sqrt[3]{z}$ **14.** _____

Additional Exercises 9.5
Form III

Name _____

Date _____

Write each as a single logarithm. Assume that variables represent positive numbers.

1. $\log_4 7 + \log_4 9$ **1.** _____

2. $\frac{1}{2}\log_7 x + \frac{1}{2}\log_7 y$ **2.** _____

3. $\log_3(x^2 - 9) - \log_3(x - 3)$ **3.** _____

4. $\log_5(4x^2 - 7) - \log_5(2x^3 + 3)$ **4.** _____

5. $\log_2 x^2 + 2\log_2 y^2 + 4\log_2 z^2$ **5.** _____

6. $5\log_4 x + \frac{1}{3}\log_4(x + 1)$ **6.** _____

7. $\frac{1}{3}\left(\log_b x + \log_b y\right) - 2\log_b(x + 3)$ **7.** _____

Write each expression as a sum or difference of multiples of logarithms.

8. $\log_2 \frac{11}{17\sqrt[5]{x}}$ **8.** _____

9. $\log_3 \frac{4\sqrt[3]{y}}{\sqrt[5]{x}}$ **9.** _____

10. $\log_5 \frac{\sqrt[3]{x}}{25y^2}$ **10.** _____

11. $\log_4 7x^2(3x + 8)$ **11.** _____

12. $\log_5 \frac{x^3 - 5x^2 + 4x}{2y^3 + y^2}$ **12.** _____

13. $\log_6 \frac{\sqrt[4]{3y}}{\sqrt{7x + 6}}$ **13.** _____

14. $\log_b \frac{\sqrt[4]{16x}\sqrt{64y}}{\sqrt[3]{27z}}$ **14.** _____

Additional Exercises 9.6
Form I

Name _____

Date _____

Use a calculator to approximate each algorithm to four decimal places.

1. $\log 11$ **1.** _____

2. $\log 1.36$ **2.** _____

3. $\ln 8$ **3.** _____

4. $\log_2 3$ **4.** _____

5. $\log_{\frac{1}{2}} 16$ **5.** _____

Solve each equation for *x*. Give an exact solution and a four-decimal place approximation.

6. $\log x = 2.7$ **6.** _____

7. $\log 4x = 1.5$ **7.** _____

8. $\log 6x = -1$ **8.** _____

9. $\ln x = -3.2$ **9.** _____

10. $\ln 7x = 3.7$ **10.** _____

11. $\log_5(3x-1) = 3$ **11.** _____

12. $\log(2x-1) = 2$ **12.** _____

13. $\log(6x-15) = 1.3$ **13.** _____

14. $\ln(7x-4) = 1$ **14.** _____

15. $\ln(x+3) = 4$ **15.** _____

Additional Exercises 9.6
Form II

Name _____

Date _____

Use a calculator to approximate each algorithm to four decimal places.

1. $\log 14$ **1.** _____

2. $\ln 0.059$ **2.** _____

3. $\log_8 11$ **3.** _____

4. $\log_{\frac{1}{2}} 12$ **4.** _____

5. $\log_7 \frac{1}{9}$ **5.** _____

Solve each equation for *x*. Give an exact solution and a four-decimal place approximation.

6. $\log x = 3.4$ **6.** _____

7. $\log 18x = 1.4$ **7.** _____

8. $\ln x = 5.1$ **8.** _____

9. $\ln 5x = 0.79$ **9.** _____

10. $\ln(7x+2) = 3.4$ **10.** _____

11. $\log(7x-5) = -3$ **11.** _____

12. $\log(4x+1) = -2.3$ **12.** _____

13. $\log(3x+1)^2 = 4$ **13.** _____

14. $\ln(3x-1) = -1.4$ **14.** _____

15. $\ln(x+4)^3 = 0.72$ **15.** _____

Additional Exercises 9.6
Form III

Name _____

Date _____

Use a calculator to approximate each algorithm to four decimal places.

1. $\log 4.62$ **1.** _____

2. $\log_3 7$ **2.** _____

3. $\ln 0.041$ **3.** _____

4. $\log_5 \frac{1}{7}$ **4.** _____

5. $\log \sqrt[3]{10}$ **5.** _____

Solve each equation for *x*. Give an exact solution and a four-decimal place approximation.

6. $\log x = 5.3$ **6.** _____

7. $\log \frac{17x}{5} = 4$ **7.** _____

8. $\ln x = -5.6$ **8.** _____

9. $\ln \frac{x}{9} = 2.7$ **9.** _____

10. $\log(7x-3) = -0.4$ **10.** _____

11. $\log\left(\frac{x}{5}\right)^2 = 1.4$ **11.** _____

12. $\log(8x+2)^3 = 5$ **12.** _____

13. $\log(100x^2+300) = 3$ **13.** _____

14. $\ln(9x-4)^2 = 0.24$ **14.** _____

15. $\ln\left(\frac{5x^2-20}{x+2}\right)^4 = 0.67$ **15.** _____

Additional Exercises 9.7
Form I

Name _____

Date _____

Solve each equation. Give an exact solution and a four-decimal place approximation.

1. $4^x = 8$ **1.** _____

2. $2^x = 12$ **2.** _____

3. $3^x = 13$ **3.** _____

4. $4^x = 15$ **4.** _____

5. $6^{x-2} = 4$ **5.** _____

Solve each equation.

6. $2\log x = \log 36$ **6.** _____

7. $\frac{1}{2}\log_3 x = 2\log_3 3$ **7.** _____

8. $\log_2(2x+1) = 3$ **8.** _____

9. $2\log_5 x = 5\log_5 2$ **9.** _____

10. $3\log_2 x = -\log_2 27$ **10.** _____

11. $\log_3(x+4) - \log_3 9 = 1$ **11.** _____

12. $\log x + \log 5 = 2$ **12.** _____

13. $\ln 6x^3 - \ln 3x^2 = \ln 4.5$ **13.** _____

14. $\log x + \log(x+1) = \log 12$ **14.** _____

15. $\ln 3x + \ln(x+4) = \ln 36$ **15.** _____

Additional Exercises 9.7
Form II

Name _____

Date _____

Solve each equation. Give an exact solution and a four-decimal place approximation.

1. $4^x = 7$ — **1.** _____

2. $2^x = 10$ — **2.** _____

3. $3^{2x} = 14$ — **3.** _____

4. $3^{x+2} = 5$ — **4.** _____

5. $7^{x-2} = 6$ — **5.** _____

Solve each equation.

6. $\log_5 x = 8\log_5 2$ — **6.** _____

7. $\log_3(3x-2) = 2$ — **7.** _____

8. $-2\log_4 x = \log_4 12$ — **8.** _____

9. $\ln 8x^2 - \ln 2x = -1.5$ — **9.** _____

10. $\log_4 x + \log_4(x-3) = 1$ — **10.** _____

11. $3\log_2(x-1) + \log_2 4 = 5$ — **11.** _____

12. $\log x + \log(x+3) = \log 10$ — **12.** _____

13. $\log_2(9x+5) - \log_2(x+2) = 3$ — **13.** _____

14. $\log_6 x + \log_6(x+9) = 2$ — **14.** _____

15. $\ln(36-x^2) - \ln(6+x) = \frac{1}{2}\ln x$ — **15.** _____

Additional Exercises 9.7
Form III

Name _____

Date _____

Solve each equation. Give an exact solution and a four-decimal place approximation.

1. $15^{4x} = 225$ **1.** _____

2. $3^{x-5} = 11$ **2.** _____

3. $5^{2x-4} = 14$ **3.** _____

4. $e^{2x} = 21$ **4.** _____

5. $e^{4x-3} = \frac{1}{7}$ **5.** _____

Solve each equation.

6. $\log_5(3x-1) = 2$ **6.** _____

7. $\frac{1}{3}\log_3 x = 2\log_3 2$ **7.** _____

8. $\log x + \log 50 = 2$ **8.** _____

9. $\log_{10} x + \log_{10}(x+15) = 2$ **9.** _____

10. $\log_2(x+2) - \log_2(x-5) = 3$ **10.** _____

11. $\log_b(x^2-9) = \log_b(x+3) + \log_b 7$ **11.** _____

12. $\ln 3x^2 - \ln 4x = -4.5$ **12.** _____

13. $\ln(1-x) - \ln(1+x) = \ln e$ **13.** _____

14. $\log_b(2x-1) = \log_b(4x-3) - \log_b x$ **14.** _____

15. $\log_b(x^2-1) = \log_b(4x^2+6x+2) - \log_b 3$ **15.** _____

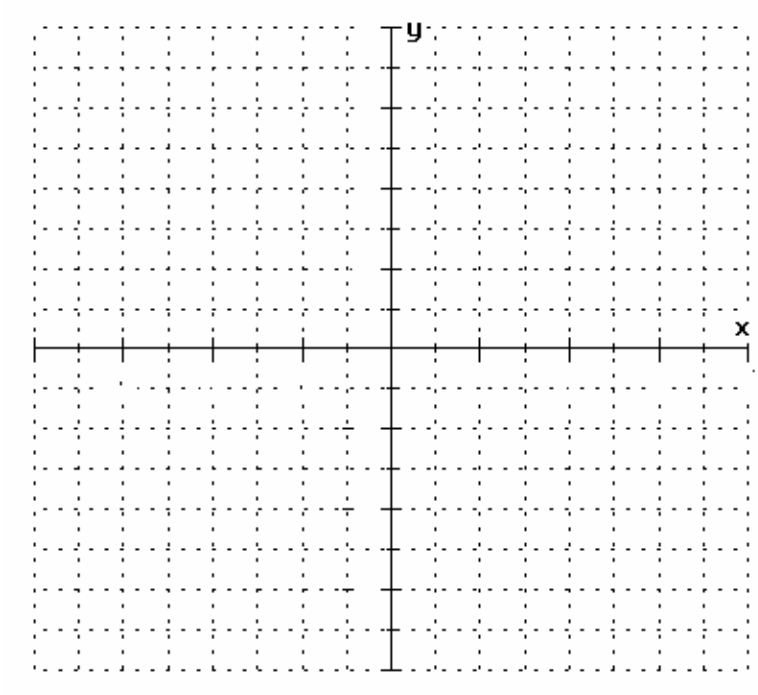
Name: **Date:**
Instructor: **Section:**

Section 9.2 Inverse Functions

Objective: Determine if a function is one-to-one, then find and graph its inverse.
Suggested Format: Small Group Format
Time: 15 minutes

Given $f(x) = 3x - 2$.

1. Determine if the function is one-to-one by graphing it. Discuss your reasoning.



2. If $f(x)$ is one-to-one, find a formula for the inverse.

3. Graph the inverse on the same set of axis as you graphed $f(x)$. Draw the line $y = x$ and compare the two graphs. What do you notice?

Name: **Date:**
Instructor: **Section:**

Section 9.7 Exponential and Logarithmic Equations and Applications

Objective: Solve problems that can be modeled by exponential functions.
Suggested Format: Small Group Format
Time: 20 minutes

In 2008, the black bear population in Granite Park was 510 and the exponential growth rate was 5% per year.

Exponential Growth Model:
$P(t) = P_o e^{kt}$ where $P(t)$ is the population at time t,
t is the number of years after initial year
P_o is the population at the initial year ($t = 0$)
k is the exponential growth rate

1. Find the exponential growth function.

2. According to this model, what will be the population in 2012? Round to the nearest whole number.

3. In what year will the bear population reach 688?

Name: **Date:**
Instructor: **Section:**

Chapter 9 Test Form A

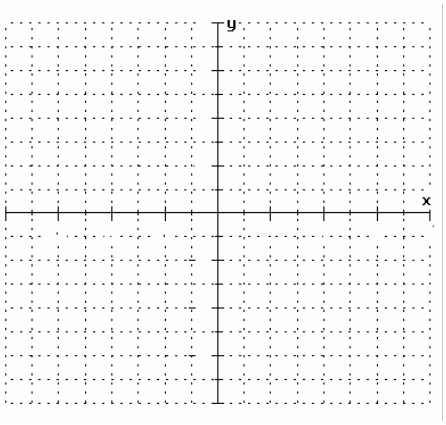
For $f(x) = 2x - 3$ and $g(x) = x^2 + 9$, find the following.

1. $(f \circ g)(x)$ **1.** _____

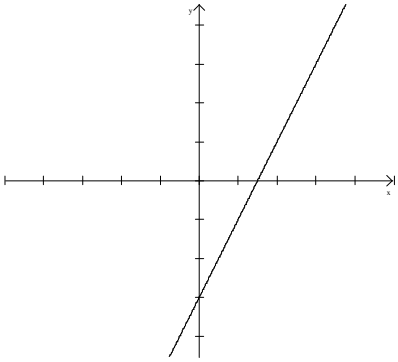
2. $(f \circ g)(2)$ **2.** _____

3. $(g \circ f)(x)$ **3.** _____

4. Graph $f(x) = x - 4$ and its inverse on the same set of axes. **4.**



5. Determine if whether the function below is one-to-one. **5.** _____



6. Is $f(x) = -3x + 5$ a one-to-one function? **6.** _____

7. Find the inverse of $f(x) = \dfrac{2}{x+3}$. **7.** _____

Name: **Date:**
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Chapter 9 Test Form A *cont'd*

8. Find the inverse of $F = \{(0, 1), (1, 2), (2, 3), (3, 4)\}$. **8.** _____

Use the properties of logarithms to write each expression as a single logarithm.

9. $\log_{12} x + \log_{12} 5$ **9.** _____

10. $2\log_9 (x+1) - \log_9 y$ **10.** _____

11. Write the expression $\log_6 \frac{3x}{y^2}$ as the sum or difference of multiples of logarithms. **11.** _____

12. If $\log_b 3 = 0.8$ and $\log_b 6 = 1.3$, find the value of $\log_b 18$. **12.** _____

13. Approximate $\log_6 22$ to four decimal places. **13.** _____

14. Solve $3^{x-2} = \frac{1}{9}$. **14.** _____

15. Solve $4^{x+5} = 6$. Approximate the solution to four decimal places. **15.** _____

16. Simplify. $-\log_5 625$ **16.** _____

Solve each logarithmic equation.

17. $\log_5 x = -3$ **17.** _____

18. $\ln e^5 = 3x$ **18.** _____

19. $\log_7 4 + \log_7 x = 3$ **19.** _____

20. $2\log x - \log 7 = \log 112$ **20.** _____

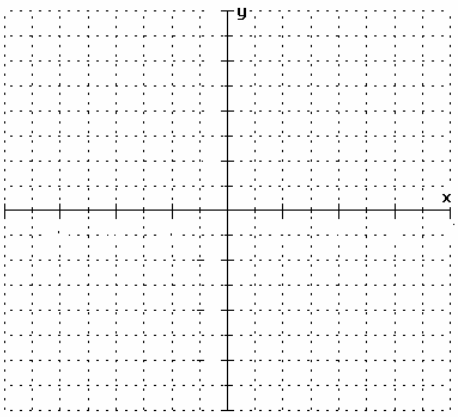
21. $\ln(5x-2) = 12$ Approximate the solution to four decimal places. **21.** _____

Name: **Date:**
Instructor: **Section:**

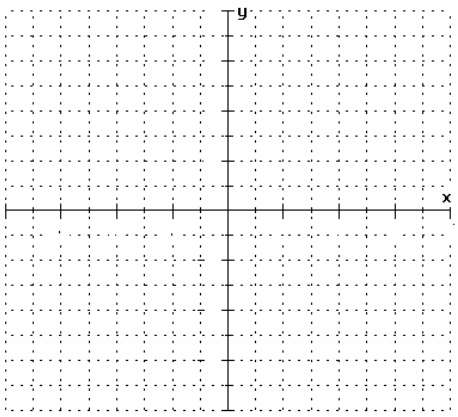
Chapter 9 Test Form A *cont'd*

Graph.

22. $y = 2^x + 3$



23. $y = \log_2 x$



24. Using the formula $A = P\left(1 + \frac{r}{n}\right)^{nt}$, find how long it takes a \$600 investment to grow to \$700 if it is invested at 8% interest compounded monthly. **24.** _____

25. Using the formula $w = 0.00185h^{2.67}$, where w is a boy's weight and h is his height in inches, estimate the height of a boy whose weight is 100 pounds. **25.** _____

Name: **Date:**
Instructor: **Section:**

Chapter 9 Test Form B

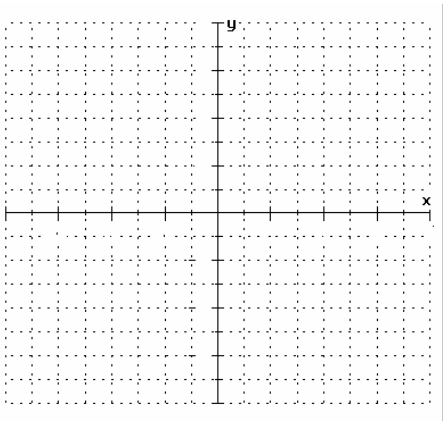
For $f(x) = 2x - 3$ and $g(x) = x^2 - 3x - 2$, find the following.

1. $(f \circ g)(x)$ **1.** _____

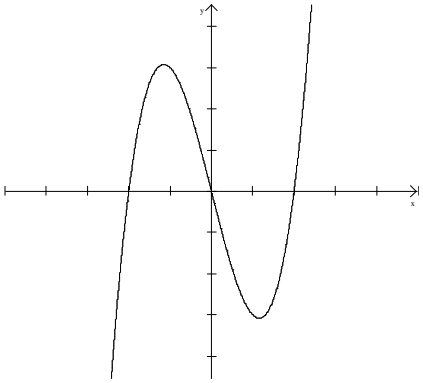
2. $(f \circ g)(-2)$ **2.** _____

3. $(g \circ f)(x)$ **3.** _____

5. Graph $f(x) = 3x - 4$ and its inverse on the same set of axes. **4.**



5. Determine if the function below is one-to-one. **5.** _____



6. Is $f(x) = x^2 - 2x + 1$ a one-to-one function? **6.** _____

7. Find the inverse of $f(x) = \sqrt{x - 2}$. **7.** _____

Name: **Date:**
Instructor: **Section:**

Chapter 9 Test Form B *cont'd*

8. Find the inverse of $F = \{(1, 2), (2, 3), (3, 4), (4, 5)\}$. **8.** _____

Use the properties of logarithms to write each expression as a single logarithm.

9. $\log_2 3 + \log_2 5$ **9.** _____

10. $\log_6 4 - 3\log_6 (y+2)$ **10.** _____

11. Write the expression $\log_4 \frac{9x^2}{y^2}$ as the sum or difference of multiples of logarithms. **11.** _____

12. Find the value of $\log_7 49$. **12.** _____

13. Approximate $\log_5 15$ to four decimal places. **13.** _____

14. Solve $256 = 2^{16x}$. **14.** _____

15. Solve $e^{3x-9} = 8$. Approximate the solution to four decimal places. **15.** _____

16. Simplify. $3\log_2 4$ **16.** _____

Solve each logarithmic equation.

18. $\log_3 x = -3$ **17.** _____

18. $\ln\left(\frac{1}{3^{\frac{1}{4}}}\right) = x$ **18.** _____

19. $\log_7 3 + \log_7 x = 4$ **19.** _____

20. $\log(x+1) - \log(x-1) = \log 15$ **20.** _____

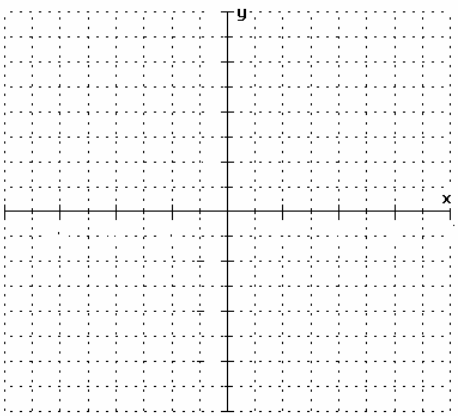
21. $\ln(3x+5) = 12$ Approximate the solution to four decimal places. **21.** _____

Name: **Date:**
Instructor: **Section:**

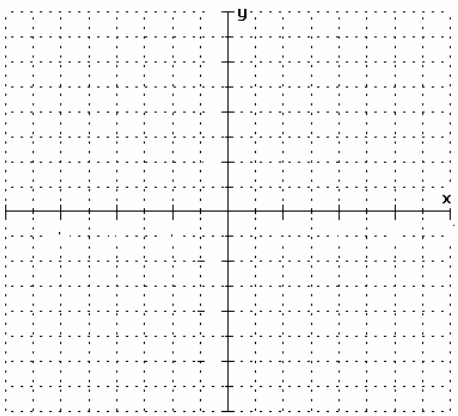
Chapter 9 Test Form B *cont'd*

Graph.

22. $y = \left(\frac{1}{3}\right)^x - 2$



23. $y = \log_5 x$



24. Using the formula $A = P\left(1 + \frac{r}{n}\right)^{nt}$, find how long it takes an investment to double if it is invested at 8% interest compounded monthly. **24.** _____

25. In a farming area, the number of acres of cropland rendered unusable by a new weed infestation is increasing at a rate of 6% per year. If 20 acres are unusable now, how many will be unusable in four years? **25.** _____

Name: **Date:**
Instructor: **Section:**

Chapter 9 Test Form C

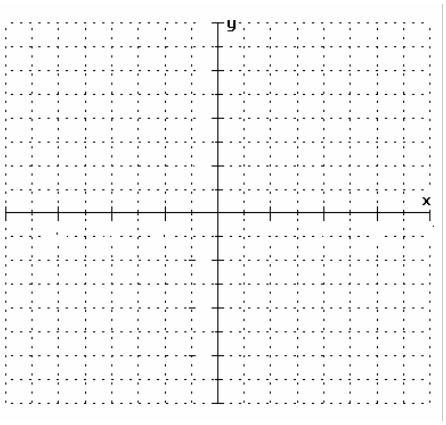
For $f(x) = 3 - x^2$ and $g(x) = 5x + 2$, find the following.

1. $(f \circ g)(x)$ **1.** _____

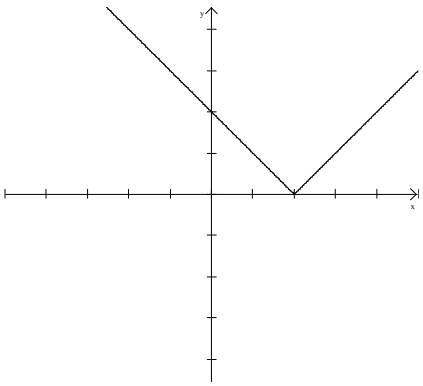
2. $(f \circ g)(-2)$ **2.** _____

3. $(g \circ f)(x)$ **3.** _____

6. Graph $f(x) = 2x - 3$ and its inverse on the same set of axes. **4.**



5. Determine if whether the function below is one-to-one. **5.** _____



6. Is $F = \{(1, 1), (2, 2), (3, 3), (4, 4)\}$ a one-to-one function? **6.** _____

7. Find the inverse of $f(x) = x^3 + 3$. **7.** _____

Name: **Date:**
Instructor: **Section:**

Chapter 9 Test Form C ***cont'd***

8. Find the inverse of **8.** _____

$$F=\left\{\left(3,\frac{1}{2}\right),\left(4,\frac{1}{3}\right),\left(5,\frac{1}{4}\right),\left(6,\frac{1}{5}\right)\right\}$$

Use the properties of logarithms to write each expression as a single logarithm.

9. $\log_2 3+\log_2 5$ **9.** _____

10. $4\log_5 x+\log_5 y-3\log_5 z$ **10.** _____

11. Write the expression $\log_3 \frac{(4-x)^2}{2y}$ as the sum or difference of multiples of logarithms. **11.** _____

12. If $\log_b 5=-1.23$ and $\log_b 2=-0.53$, find the value of $\log_b \frac{5}{2}$. **12.** _____

13. Approximate $\log_7 14$ to four decimal places. **13.** _____

14. Solve $81=9^{3x-1}$. **14.** _____

15. Solve $4^{x+6}=9$. Approximate the solution to four decimal places. **15.** _____

16. Simplify. $-2\log_3 \frac{1}{81}$ **16.** _____

Solve each logarithmic equation.

17. $\log_6 x=-3$ **17.** _____

18. $\ln e^3=x$ **18.** _____

19. $\log_7 4-\log_7 x=-2$ **19.** _____

20. $\log(x+2)+\log(x-2)=\log 5$ **20.** _____

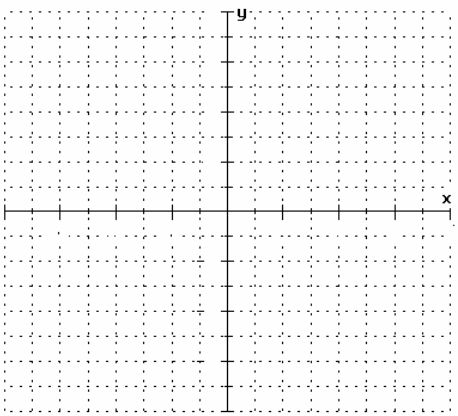
21. $\ln(2x+3)=2$ Approximate the solution to four decimal places. **21.** _____

Name: **Date:**
Instructor: **Section:**

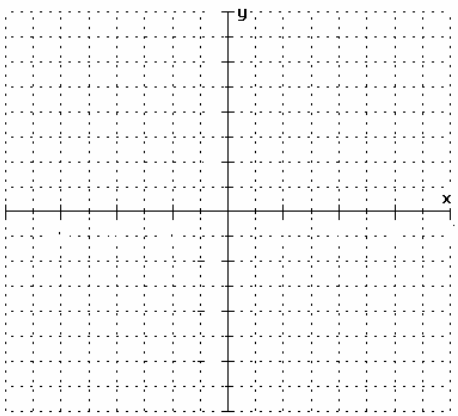
Chapter 9 Test Form C *cont'd*

Graph.

22. $y = -\left(\frac{1}{2}\right)^x - 3$



23. $y = \log_3 x - 2$



24. Using the formula $A = P\left(1 + \frac{r}{n}\right)^{nt}$, find how long it takes a \$900 investment to grow to \$1000 if it is invested at 8.5% interest compounded weekly. **24.** _____

25. Using the formula $P = 14.7e^{-0.21x}$, where P is the atmospheric pressure in pounds per square inch and x is the elevation above sea level in miles, find the elevation above sea level of a place where the atmospheric pressure is 10.73 pounds per square inch. **25.** _____

Name: **Date:**
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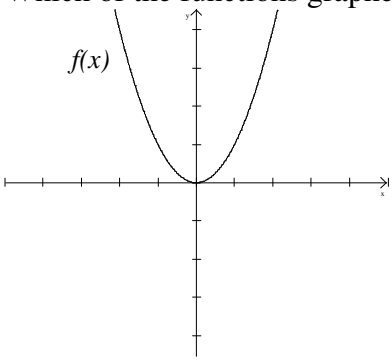
Chapter 9 Test Form D

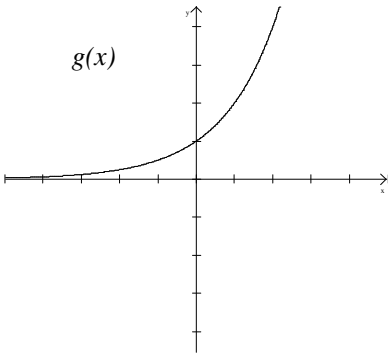
Circle the correct answer.

1. For $f(x) = x - 3$ and $g(x) = x^2 + 2$, find $(f \circ g)(4)$.

a. 10 **b.** 13 **c.** 15 **d.** 18

2. Which of the functions graphed is a one-to-one function?





a. $f(x)$ only **b.** $g(x)$ only **c.** neither **d.** both

3. Which of the relations shown below is a one-to-one function?
$F = \{(1, 2), (2, 1), (3, 2), (4, 3)\}$
$G = \{(1, 6), (2, 5), (5, 2), (6, 1)\}$

a. F only **b.** G only **c.** neither **d.** both

4. Find the inverse of the one-to-one function $f(x) = 3x - 5$.

a. $f^{-1}(x) = 3x + 5$ **b.** $f^{-1}(x) = -3x + 5$
c. $f^{-1}(x) = \dfrac{x+5}{3}$ **d.** $f^{-1}(x) = \dfrac{x-5}{3}$

5. Find the inverse of the one-to-one function $f(x) = x^3 - 2$.

a. $f^{-1}(x) = \sqrt[3]{x-2}$ **b.** $f^{-1}(x) = \sqrt[3]{x+2}$
c. $f^{-1}(x) = \sqrt[3]{x} + 2$ **d.** $f^{-1}(x) = \sqrt[2]{x} - 2$

Name: **Date:**
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Chapter 9 Test Form D *cont'd*

6. Find the inverse of the one-to-one function $f(x) = 2\log_7 x$.

a. $f^{-1}(x) = 2^{\frac{7}{x}}$ **b.** $f^{-1}(x) = 7^{\frac{x}{2}}$ **c.** $f^{-1}(x) = x^{\frac{7}{2}}$ **d.** $f^{-1}(x) = 7x^2$

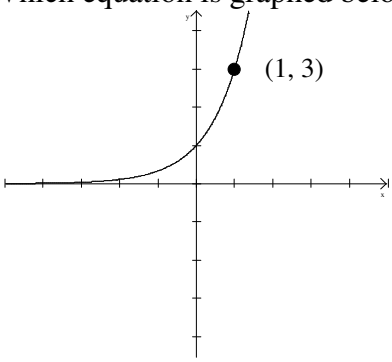
7. Simplify the expression $\log_{\frac{2}{3}} \frac{9}{4}$.

a. $-\frac{1}{2}$ **b.** -2 **c.** $\frac{1}{2}$ **d.** 2

8. Simplify the expression $\log 10 + \log_6 36$

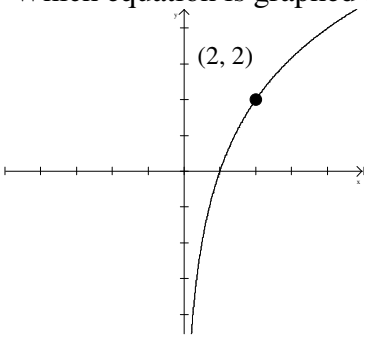
a. -3 **b.** 3 **c.** 6 **d.** -6

9. Which equation is graphed below?



a. $y = \left(\frac{1}{3}\right)^x$ **b.** $y = x^3$ **c.** $y = x^{\frac{1}{3}}$ **d.** $y = 3^x$

10. Which equation is graphed below?



a. $y = \log_2 x$ **b.** $y = \log_4 x$ **c.** $y = 2\log_2 x$ **d.** $y = \frac{1}{2}\log_4 x$

Chapter 9 Test Form D *cont'd*

11. Write $\log_9 5 - 3\log_9 7$ as a single logarithm.

a. $\log_9 338$ **b.** $\log_9 \frac{343}{338}$ **c.** $\log_9 \frac{338}{5}$ **d.** $\log_9 \frac{5}{343}$

12. Write $\log_4 \frac{7x^2}{3}$ as a sum or difference of multiples of logarithms.

a. $\log_4 3 - 2\log_4 x + \log_4 7$ **b.** $\log_4 7x^2 + \log_4 3$
c. $\log_4 7 + 2\log_4 x - \log_4 3$ **d.** $\log_4 3x^2 - \log_4 7$

Solve each equation.

13. $32 = 2^{5x+2}$

a. $-\frac{5}{3}$ **b.** $\frac{3}{5}$ **c.** $-\frac{3}{5}$ **d.** $\frac{5}{3}$

14. $\left(\frac{1}{4}\right)^{x^2+1} = 16^x$

a. $-1, 1$ **b.** 1 **c.** -1 **d.** $\varnothing$

15. $\log_9 x = \frac{1}{2}$

a. 9 **b.** 81 **c.** 3 **d.** $\frac{9}{2}$

16. $\log_x 81 = -2$

a. 9 **b.** -9 **c.** $-\frac{1}{9}$ **d.** $\frac{1}{9}$

17. $\log_2(x-1) - \log_2(x+4) = -2$

a. $\frac{5}{3}$ **b.** $\frac{7}{3}$ **c.** $\frac{8}{3}$ **d.** 3

Name: **Date:**
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Chapter 9 Test Form D *cont'd*

18. $\ln(2x-1) = -0.2$ Approximate the solution to four decimal places.

a. 0.8347 **b.** 0.9094 **c.** 1.8347 **d.** 1.9094

19. $3^{2x-1} = 27$

a. 9 **b.** 1 **c.** 2 **d.** 3

20. Find the inverse of $F = \left\{\left(\frac{1}{10},10\right),\left(\frac{1}{2},2\right),(1,1),\left(2,\frac{1}{2}\right),\left(10,\frac{1}{10}\right)\right\}$

a. $F^{-1} = \left\{\left(10,\frac{1}{10}\right),\left(2,\frac{1}{2}\right),(1,1),\left(\frac{1}{2},2\right),\left(\frac{1}{10},10\right)\right\}$

b. $F^{-1} = \left\{\left(-10,\frac{1}{10}\right),\left(-2,\frac{1}{2}\right),(-1,1),\left(-\frac{1}{2},2\right),\left(-\frac{1}{10},10\right)\right\}$

c. $F^{-1} = \left\{\left(-10,-\frac{1}{10}\right),\left(-2,-\frac{1}{2}\right),(-1,-1),\left(-\frac{1}{2},-2\right),\left(-\frac{1}{10},-10\right)\right\}$

d. $F^{-1} = \left\{\left(-\frac{1}{10},-10\right),\left(-\frac{1}{2},-2\right),(-1,-1),\left(-2,-\frac{1}{2}\right),\left(-10,-\frac{1}{10}\right)\right\}$

21. If $\log_b 4 = 0.63$ and $\log_b 7 = 0.89$, find $\log_b \frac{49}{4}$

a. 1.15 **b.** 0.26 **c.** 1.52 **d.** 2.16

22. Approximate $\log_{\frac{3}{2}} 9$ to four decimal places.

a. 2.3980 **b.** 2.7095 **c.** 5.4190 **d.** 5.9845

Chapter 9 Test Form D *cont'd*

23. The temperature of an overheating engine is increasing exponentially, according to the formula $T = T_0 (2.7)^{0.14t}$ where t is in seconds. To the nearest tenth of a second, how long will it take for the temperature to double?

a. 3.5 seconds **b.** 2.0 seconds **c.** 4.5 seconds **d.** 5.0 seconds

24. Using the formula $A = P\left(1+\frac{r}{n}\right)^{nt}$, find how long it takes a \$1200 investment to earn \$180 interest if it is invested at 7.5% interest compounded semiannually.

a. 1.5 years **b.** 2.5 years **c.** 2 years **d.** 3 years

25. An experiment teaching rats to run mazes shows that a typical rat can remember a maximum of 23 turns. Find how many weeks it should take a typical rat to learn 15 turns, using the learning curve formula $t = 5\ln\left(\frac{23}{23-N}\right)$ where N is the number of weeks it should take to learn t turns.

a. 4.8 weeks **b.** 4.0 weeks **c.** 21.9 weeks **d.** 6.8 weeks

Name: **Date:**
Instructor: **Section:**

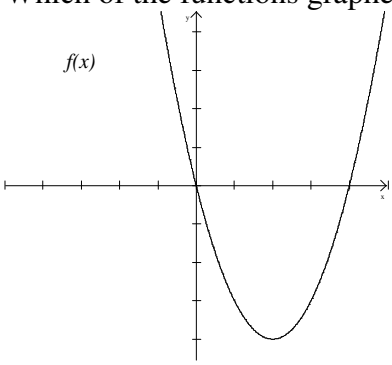
Chapter 9 Test Form E

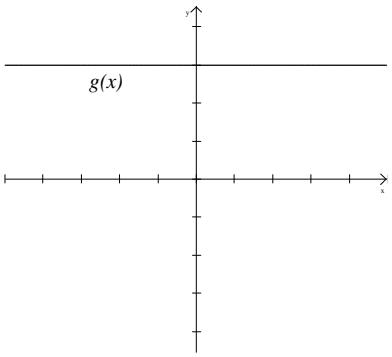
Circle the correct answer.

1. For $f(x) = 3 - x^2$ and $g(x) = 2x - 3$, find $(f \circ g)(-2)$.

a. 46 **b.** –46 **c.** –5 **d.** 5

2. Which of the functions graphed is a one-to-one function?





a. $f(x)$ only **b.** $g(x)$ only **c.** neither **d.** both

3. Which of the relations shown below is a one-to-one function?
$H = \{(3, 6), (6, 3), (6, 2), (2, 3)\}$
$K = \{(-1, -1), (-2, -3), (-3, -4), (-4, -5)\}$

a. H only **b.** K only **c.** neither **d.** both

4. Find the inverse of the one-to-one function $f(x) = 3x - 7$.

a. $f^{-1}(x) = \dfrac{x-7}{3}$ **b.** $f^{-1}(x) = -3x - 7$

c. $f^{-1}(x) = \dfrac{x+7}{3}$ **d.** $f^{-1}(x) = 3x + 7$

5. Find the inverse of the one-to-one function $f(x) = (x-5)^3 + 8$.

a. $f^{-1}(x) = x - 3$ **b.** $f^{-1}(x) = \sqrt[3]{x} - 117$

c. $f^{-1}(x) = \sqrt[3]{x-8} + 5$ **d.** $f^{-1}(x) = \sqrt[2]{x+8} - 5$

Name: **Date:**
Instructor: **Section:**

Chapter 9 Test Form E *cont'd*

6. Find the inverse of the one-to-one function $f(x) = 3\log_3 x$.

a. $f^{-1}(x) = 3^{\frac{x}{3}}$ **b.** $f^{-1}(x) = x^{\frac{3}{x}}$ **c.** $f^{-1}(x) = x^3$ **d.** $f^{-1}(x) = \frac{1}{x}$

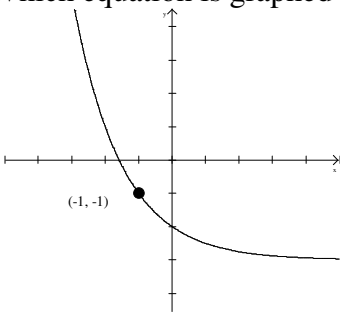
7. Simplify the expression $\log_{\frac{3}{2}} \frac{8}{27}$.

a. -3 **b.** 3 **c.** $-\frac{1}{3}$ **d.** $\frac{1}{3}$

8. Simplify the expression $\log_3 81 - \log_3 1$

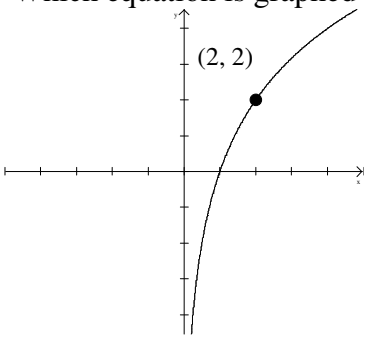
a. 1 **b.** 3 **c.** 4 **d.** 2

9. Which equation is graphed below?



a. $y = -\left(\frac{1}{2}\right)^x - 3$ **b.** $y = \left(\frac{1}{2}\right)^x - 3$ **c.** $y = -2^x - 3$ **d.** $y = (-3)^x - \frac{1}{2}$

10. Which equation is graphed below?



a. $y = \log_{\frac{1}{2}} x$ **b.** $y = \frac{1}{2}\log_{\frac{1}{2}} x$ **c.** $y = 2\log_2 x$ **d.** $y = \frac{1}{2}\log_4 x$

Name: **Date:**
Instructor: **Section:**

Chapter 9 Test Form E ***cont'd***

11. Write $\log_6 5 + 3\log_6 \frac{2}{3}$ as a single logarithm.

a. $\log_6 10$ **b.** $\log_6 \frac{20}{27}$ **c.** $\log_6 \frac{40}{27}$ **d.** $\log_6 \frac{40}{3}$

12. Write $\log_4 \frac{7}{5x^2}$ as a sum of difference of multiples of logarithms.

a. $\log_4 7 - 2\log_4 5x$ **b.** $\log_4 7 - 2\log_4 5x$
c. $\log_4 7 - \log_4 5 + 2\log_4 x$ **d.** $\log_4 7 - \log_4 5 - 2\log_4 x$

Solve each equation.

13. $3 = \left(\frac{1}{3}\right)^{2x}$

a. -2 **b.** $-\frac{1}{2}$ **c.** 3 **d.** $\frac{1}{3}$

14. $\left(\frac{1}{49}\right)^{-1} = 7^x$

a. 2 **b.** -2 **c.** 1 **d.** $\frac{1}{2}$

15. $\log_{\frac{1}{2}} x = -3$

a. $\frac{1}{8}$ **b.** 8 **c.** $-\frac{1}{8}$ **d.** -8

16. $\log_x \frac{1}{125} = 3$

a. 5 **b.** $\frac{1}{5}$ **c.** -5 **d.** $-\frac{1}{5}$

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Chapter 9 Test Form E *cont'd*

17. $\log_3(x-1)-\log_3(x-5)=1$

a. 5 **b.** 6 **c.** 7 **d.** 8

18. $\ln(x-4)=1.5$ Approximate the solution to four decimal places.

a. 7.1835 **b.** 8.4817 **c.** 9.2462 **d.** 9.3154

19. $2^{3x-5}=17$ Approximate the solution to four decimal places.

a. 2.9463 **b.** 2.9871 **c.** 3.0292 **d.** 3.2775

20. Find the inverse of $F=\left\{\left(\frac{1}{4},4\right),\left(\frac{1}{8},8\right),(1,1),\left(8,\frac{1}{8}\right),\left(4,\frac{1}{4}\right)\right\}$

a. $F^{-1}=\left\{\left(4,\frac{1}{4}\right),\left(8,\frac{1}{8}\right),(1,1),\left(\frac{1}{8},8\right),\left(\frac{1}{4},4\right)\right\}$

b. $F^{-1}=\left\{\left(-4,\frac{1}{4}\right),\left(-8,\frac{1}{8}\right),(-1,1),\left(-\frac{1}{8},8\right),\left(-\frac{1}{4},4\right)\right\}$

c. $F^{-1}=\left\{\left(-4,-\frac{1}{4}\right),\left(-8,-\frac{1}{8}\right),(-1,-1),\left(-\frac{1}{8},-8\right),\left(-\frac{1}{4},-4\right)\right\}$

d. $F^{-1}=\left\{\left(-\frac{1}{4},-4\right),\left(-\frac{1}{8},-8\right),(-1,-1),\left(-8,-\frac{1}{8}\right),\left(-4,-\frac{1}{4}\right)\right\}$

21. If $\log_a 10=b$ and $\log_a 5=c$, find $\log_a 2$

a. $b-c$ **b.** $b+2c$ **c.** $2(b+c)$ **d.** $b-c$

22. Approximate $\log_{\frac{2}{7}}10$ to four decimal places.

a. -3.5000 **b.** 0.2857 **c.** -1.8380 **d.** -0.5441

Chapter 9 Test Form E *cont'd*

23. Using the formula $A = P\left(1+\frac{r}{n}\right)^{nt}$, find amount owed at the end of 4 years if \$300 is loaned at a rate of 12% compounded monthly.

a. \$483.67 **b.** \$481.41 **c.** \$478.15 **d.** \$446.81

24. Find the atmospheric pressure P o\in psi on top of Mt. Koscinsko, 1.4 miles above sea level, using the formula $P = 14.7e^{-0.21x}$ where x is the distance above sea level in miles.

a. 1.39 psi **b.** 5.13 psi **c.** 10.96 psi **d.** 13.77 psi

25. Using the formula $A = P\left(1+\frac{r}{n}\right)^{nt}$, find how long it takes a \$1300 investment to earn \$135 interest if it is invested at 6.85% interest compounded quarterly.

a. 9 months **b.** 1 year **c.** $1\frac{1}{2}$ years **d.** 2 years

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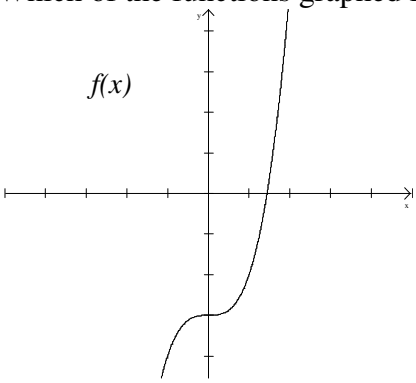
Chapter 9 Test Form F

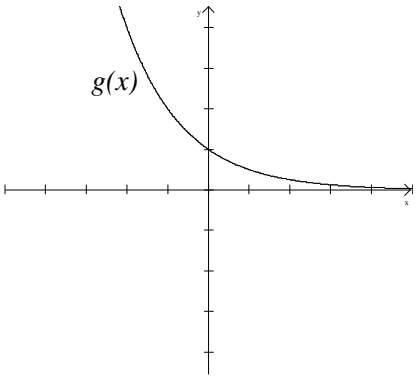
Circle the correct answer.

1. For $f(x) = x^2 - 2x$ and $g(x) = x - 3$, find $(f \circ g)(2)$.

a. -15 **b.** 3 **c.** 15 **d.** -3

2. Which of the functions graphed is a one-to-one function?





a. $f(x)$ only **b.** $g(x)$ only **c.** neither **d.** both

3. Which of the relations shown below is a one-to-one function?
$F = \{(9, 7), (7, -7), (9, 9), (-7, -9)\}$
$G = \{(2, 4), (4, 2), (-4, -4), (-2, -2)\}$

a. F only **b.** G only **c.** neither **d.** both

4. Find the inverse of the one-to-one function $f(x) = \frac{1}{2}x + 4$.

a. $f^{-1}(x) = 2x - 8$ **b.** $f^{-1}(x) = 2x - 4$
c. $f^{-1}(x) = \frac{1}{2}x - 4$ **d.** $f^{-1}(x) = -\frac{1}{2}x - 4$

5. Find the inverse of the one-to-one function $f(x) = (x+1)^3 - 2$.

a. $f^{-1}(x) = \sqrt[3]{x+2} - 1$ **b.** $f^{-1}(x) = \sqrt[3]{x} - 1$
c. $f^{-1}(x) = \sqrt[3]{x-2} + 1$ **d.** $f^{-1}(x) = \sqrt[2]{x-1} + -2$

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Chapter 9 Test Form F *cont'd*

6. Find the inverse of the one-to-one function $f(x)=\frac{1}{3}\log_5 x$.

a. $f^{-1}(x)=5^{x+\frac{1}{3}}$ **b.** $f^{-1}(x)=3\cdot 5^x$ **c.** $f^{-1}(x)=5^{3x}$ **d.** $f^{-1}(x)=\frac{5^x}{3}$

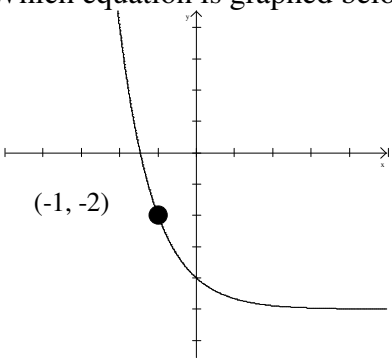
7. Simplify the expression $\log_{\frac{3}{2}}\left(\frac{16}{81}\right)$.

a. 4 **b.** –4 **c.** $\frac{1}{4}$ **d.** $-\frac{1}{4}$

8. Simplify the expression $\log_3 81+\log_3 27$

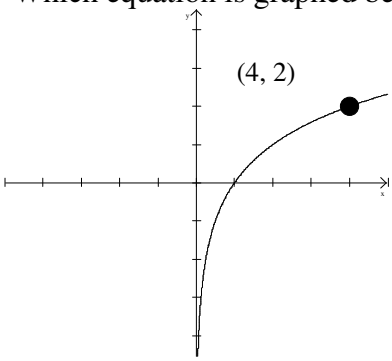
a. 8 **b.** 12 **c.** 7 **d.** 4

9. Which equation is graphed below?



a. $y=-\left(\frac{1}{3}\right)^x-5$ **b.** $y=\left(\frac{1}{3}\right)^x-5$ **c.** $y=-3^x-5$ **d.** $y=3^x-5$

10. Which equation is graphed below?



a. $y=\frac{1}{2}\log_2 x$ **b.** $y=2\log_4 x$ **c.** $y=3\log_4 x$ **d.** $y=\frac{1}{3}\log_4 x$

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Chapter 9 Test Form F *cont'd*

11. Write $4\log_{11}5-5\log_{11}3$ as a single logarithm.

a. $\log_{11}\frac{625}{243}$ **b.** $\log_{11}382$ **c.** $\log_{11}225$ **d.** $\log_{11}118$

12. Write $\log_4\frac{3y}{x+1}$ as a sum of difference of multiples of logarithms.

a. $3\log_4 y+2\log_4(x+1)$ **b.** $3\log_4 3-\log_4 y-\log_4(x+1)$

c. $\log_4 3-\log_4 y+2\log_4(x+1)$ **d.** $\log_4 3+\log_4 y-\log_4(x+1)$

Solve each equation.

13. $243^x=9^{x-2}$

a. $-\frac{4}{3}$ **b.** $-\frac{1}{2}$ **c.** $-\frac{3}{4}$ **d.** -1

14. $16^{x^2-5}=4^{8x}$

a. $-1, 5$ **b.** $-5, -1$ **c.** $-5, 1$ **d.** $1, 5$

15. $\log_5 22x=3$

a. $\frac{15}{22}$ **b.** $\frac{22}{15}$ **c.** $\frac{22}{125}$ **d.** $\frac{125}{22}$

16. $\log_x\frac{2}{3}=1$

a. $\frac{2}{3}$ **b.** $\frac{3}{2}$ **c.** 1 **d.** $-\frac{2}{3}$

17. $\log_5 8x+\log_5 2x=2$

a. 2 **b.** $-\frac{5}{4},\frac{5}{4}$ **c.** $\pm\sqrt{2}$ **d.** $\frac{\sqrt{10}}{4}$

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Chapter 9 Test Form F *cont'd*

18. $\ln(4-x)=1.5$ Approximate the solution to four decimal places.

a. -0.5712 **b.** -0.4817 **c.** 0.2247 **d.** 0.3477

19. $4^{5-2x}=29$

a. 1.2855 **b.** 1.3641 **c.** 1.4208 **d.** 1.5777

20. The graphs of a function and its inverse are symmetric about which line?

a. $y=0$ **b.** $x=0$ **c.** $y=x$ **d.** none of these

21. If $\log_b 8=8$ and $\log_b 56=A$, find $\log_b \frac{1}{56}$.

a. $8+A$ **b.** $-A$ **c.** $\frac{1}{A}$ **d.** $8+\frac{1}{A}$

22. Approximate $\log_{298} 111$ to four decimal places.

a. 0.4536 **b.** 20.6048 **c.** 0.8267 **d.** 1.2097

23. If a state park's deer population is growing at the rate of 5% annually, find how many deer there will be in that park in 8 years, if the current population is 27.

a. 35 **b.** 40 **c.** 45 **d.** 50

24. Using the formula $A=P\left(1+\frac{r}{n}\right)^{nt}$, find the minimum interest rate, compounded quarterly, needed for \$1000 to earn \$120 in one year.

a. 9.7% **b.** 11.5% **c.** 13.3% **d.** 14.0%

25. The formula $t=10\ln\left(\frac{80}{80-n}\right)$, where n is the score and t is the time in weeks, describes a dog's score on an obedience test after t weeks. Determine how long it will take a dog to score 50 on an obedience test.

a. 3.5 weeks **b.** 4.9 weeks **c.** 6.3 weeks **d.** 9.8 weeks