Mini-Lecture 10.1

The Parabola and the Circle

Learning Objectives:

1. Graph parabolas of the form $x = a(y-k)^2 + h$ and $y = a(x-h)^2 + k$.
2. Graph circles of the form $(x-h)^2 + (y-k)^2 = r^2$.
3. Write the equation of a circle given its center and radius.
4. Find the center and radius of a circle, given its equation.
5. Key vocabulary: *radius, standard form, axis of symmetry.*

Examples:

1. The graph of each equation is a parabola. Find the vertex of the parabola, and then graph it.

   a) $y = x^2$ b) $y = x^2 + 2$ c) $y = (x-2)^2$ d) $y = -2(x+1)^2 - 1$

   e) $x = y^2$ f) $x = \frac{1}{2}y^2$ g) $x = -3y^2$ h) $x = (y-2)^2 + 1$

   i) $x = -2(y+3)^2 - 3$ j) $y = x^2 - 4x + 1$ k) $x = -2y^2 + 12y - 8$

2. Graph circles of the form $(x-h)^2 + (y-k)^2 = r^2$ by determining the center and radius.

   a) $x^2 + y^2 = 9$ b) $(x-2)^2 + y^2 = 16$ c) $(x+3)^2 + (y-4)^2 = 25$

3. Write an equation of the circle with the given center and radius.

   a) $(3, 5)\,;\, 2$ b) $(-2, 4)\,;\, \sqrt{3}$ c) *the origin* $;\, 5\sqrt{2}$

4. Rewrite each equation in standard form, and determine the center and radius of each circle.

   a) $x^2 + y^2 + 6x - 4y = 23$ b) $x^2 + y^2 - 12x - 2y - 27 = 0$

Teaching Notes:

- Most students need to be reminded of how to graph vertical parabolas.
- Some students find horizontal parabolas very confusing.
- Encourage students to identify the axis of symmetry when graphing parabolas and to plot a couple of points to the right and to the left of the axis of symmetry.
- Most students understand the circle equation once they see how it results from the distance formula.
- Many students need to be reminded of the procedure for completing the square.
- Refer students to the ***Parabolas*** and ***Circle*** charts in the text.

Answers: (graph answers at end of mini-lectures) 1a) (0,0), b) (0,2), c) (2,0), d) (-1,-1), e) (0,0), f) (0,0), g) (0,0), h) (1,2), i) (-3,-3), j) (2,-3), k) (10,3); 2a) (0,0), r=3, b) (2,0), r=4, c) (-3,4), r=5; 3a) $(x-3)^2+(y-5)^2=4$, b) $(x+2)^2+(y-4)^2=3$, c) $x^2+y^2=50$; 4a) $(x+3)^2+(y-2)^2=36$, center (-3,2), r=6, b) $(x-6)^2+(y-1)^2=64$, center (6,1), r=8

Mini-Lecture 10.2

The Ellipse and the Hyperbola

Learning Objectives:

1. Define and graph an ellipse.
2. Define and graph a hyperbola.
3. Mixed practice.
4. Key vocabulary: *focus, foci, center, standard form, asymptote.*

Examples:

1. Graph each ellipse.

   a) $\frac{x^2}{9}+\frac{y^2}{16}=1$ b) $\frac{x^2}{25}+\frac{y^2}{4}=1$ c) $\frac{x^2}{16}+y^2=1$

   d) $25x^2+4y^2=100$ e) $\frac{(x+3)^2}{36}+\frac{(y-2)^2}{16}=1$ f) $\frac{(x+2)^2}{25}+\frac{(y+4)^2}{9}=1$

2. Graph each hyperbola.

   a) $\frac{x^2}{4}-\frac{y^2}{4}=1$ b) $\frac{y^2}{4}-\frac{x^2}{4}=1$ c) $\frac{x^2}{25}-\frac{y^2}{9}=1$

   d) $4y^2-x^2=16$ e) $25x^2-4y^2=100$

3. Identify each equation as that of an ellipse or a hyperbola, then sketch the graph.

   a) $\frac{x^2}{25}=1-y^2$ b) $4x^2-25y^2=100$ c) $4(x+3)^2+9(y-3)^2=36$

Teaching Notes:

- Some students understand the graphs better if the domains of 1a) and 2a) are discussed before they are graphed.
- Encourage students to memorize the standard forms of the equations of an ellipse or hyperbola centered at the origin. Then the equation for an ellipse centered at (*h, k)* can easily be remembered using graph-shifting ideas.
- Most students need to see many examples of hyperbola graphs in order to master this section.
- Students view the asymptotes as less mysterious if they are shown how a hyperbola equation behaves for very large *x* (or *y*) values. For example:
  $\frac{y^2}{b^2}-\frac{x^2}{a^2}=1 \rightarrow y=\pm\frac{b}{a}\sqrt{x^2+a^2} \rightarrow$ as *x* gets large $\rightarrow y=\pm\frac{b}{a}x$
- Refer students to the ***Ellipse with Center (0,0)*** and ***Hyperbola with Center (0,0)*** charts in this section, and the ***Conic Sections*** chart in the Integrated Review at the end of this section.

Answers: (graph answers at end of mini-lectures) 3a) $\frac{x^2}{25}+y^2=1$*, ellipse, b)* $\frac{x^2}{25}-\frac{y^2}{4}=1$*, hyperbola,*

c) $\frac{(x+3)^2}{9}+\frac{(y-3)^2}{4}=1$*, ellipse*

Mini-Lecture 10.3

Solving Nonlinear Systems of Equations

Learning Objectives:

1. Solve a nonlinear system by substitution.
2. Solve a nonlinear system by elimination.

Examples:

1. Solve each nonlinear system of equations by substitution.

a) $\begin{aligned} x^2 + y^2 &= 25 \\ x + y &= 7 \end{aligned}$ b) $\begin{aligned} y &= x^2 - 4x + 4 \\ x + y &= 14 \end{aligned}$ c) $\begin{aligned} x + y &= -3 \\ y^2 - x^2 &= 3 \end{aligned}$

2. Solve each nonlinear system of equations by elimination.

a) $\begin{aligned} x^2 + y^2 &= 52 \\ x^2 - y^2 &= 20 \end{aligned}$ b) $\begin{aligned} y &= x^2 + 2 \\ y &= -x^2 + 8 \end{aligned}$ c) $\begin{aligned} x^2 + y^2 &= 25 \\ y &= \frac{1}{5}x^2 - 5 \end{aligned}$

Teaching Notes:

- Most students understand this section better if they make a rough sketch of each system before trying to solve it.
- Encourage students to check if their intersection points agree with what the sketch suggested for the number of and the rough positions of intersection points.
- Encourage students to write all of the standard form equations for conic sections on an index card for easy reference.
- Most students have a preferred method of solving systems, either substitution or elimination. Encourage them to master both methods so that they can choose the method that is most appropriate for each situation.

Answers: 1a) {(4,3),(3,4)}, b) {(5,9),(-2,16)}, c) {(-1,-2)}; 2a) {(6,4),(6,-4),(-6,4),(-6,-4)}, b) $\{(\sqrt{3},5),(-\sqrt{3},5)\}$, *c) {(0,-5),(5,0),(-5,0)}; 3a)* $\{(\sqrt{6},3),(\sqrt{6},-3),(-\sqrt{6},3),(-\sqrt{6},-3)\}$, *b) {(4,12),(-12,-4)}, c)* $\varnothing$, *d) {(0,-1)}, e) {(-5,0),(4,3),(4,-3)}, f)* $\{(\sqrt{2},\sqrt{3}),(-\sqrt{2},\sqrt{3}),(\sqrt{2},-\sqrt{3}),(-\sqrt{2},-\sqrt{3})\}$ *; 4) 7 cm by 15 cm*

Mini-Lecture 10.4

Nonlinear Inequalities and Systems of Inequalities

Learning Objectives:

1. Graph a nonlinear inequality.
2. Graph a system of nonlinear inequalities.
3. Key vocabulary: *test region, solution region.*

Examples:

1. Graph each inequality.

a) $y > -x^2$ b) $y \le (x-5)^2 - 1$ c) $x^2 + y^2 \ge 9$

d) $\frac{x^2}{25} + \frac{y^2}{16} < 1$ e) $\frac{y^2}{9} - \frac{x^2}{4} \ge 1$ f) $y \le x^2 - 3x - 4$

2. Graph each system.

a) $\begin{cases} y > x^2 \\ 4x + 7y \le 28 \end{cases}$ b) $\begin{cases} x^2 + y^2 \le 16 \\ x^2 + y^2 \ge 4 \end{cases}$ c) $\begin{cases} x^2 + y^2 \le 81 \\ 6x + 4y \le 24 \end{cases}$

d) $\begin{cases} x - y \le -1 \\ \frac{y^2}{4} - \frac{x^2}{25} \le 1 \end{cases}$ e) $\begin{cases} x^2 + y^2 \le 16 \\ x < y^2 \end{cases}$ f) $\begin{cases} x^2 + y^2 \le 25 \\ \frac{x^2}{9} + \frac{y^2}{64} \ge 1 \end{cases}$

Teaching Notes:

- Some students need to be reminded of how to graph systems of linear inequalities before attempting systems of nonlinear inequalities.
- Remind students to always check test points.
- Some students have trouble understanding where the solution region is for nonlinear inequalities. It often helps them to discuss when less than means "below" or "within" a graph, and when greater than means "above" or "outside" of a graph.

Answers: (graph answers at end of mini-lectures)

Additional Exercises 10.1
Form I

Name _____

Date _____

Find the center and radius of the following circles.

1. $(x - 6)^2 + (y - 2)^2 = 9$ **1.** _____

2. $(x - 3)^2 + (y + 5)^2 = 4$ **2.** _____

3. $x^2 + y^2 + 2x - 6y + 1 = 0$ **3.** _____

Find the vertex of the following parabolas.

4. $y = -(x - 1)^2 - 2$ **4.** _____

5. $y = 3(x + 2)^2 - 9$ **5.** _____

6. $y = x^2 - 5x + 33$ **6.** _____

Write an equation of the circle with the given center and radius.

7. $(3, 8)$; $r = 4$ **7.** _____

8. $(7, -2)$; $r = 5$ **8.** _____

9. $(-4, -6)$; $r = 11$ **9.** _____

Write an equation of the parabola as a function of x or y with the given vertex and value for a. Determine whether the parabola opens upward, downward, to the left, or to the right.

10. $f(x)$ with vertex $(0, -6)$; $a = 4$ **10.** _____

11. $g(y)$ with vertex $(4, 7)$; $a = -6$ **11.** _____

12. $g(y)$ with vertex $(-9, -6)$; $a = 11$ **12.** _____

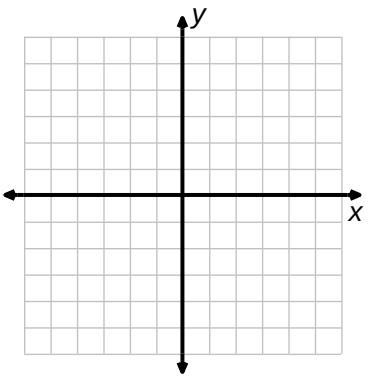
Additional Exercises 10.1 (*cont.*)

Name _____

Sketch the graph of each equation. If the graph is a parabola, find its vertex. If the graph is a circle, find its center and radius.

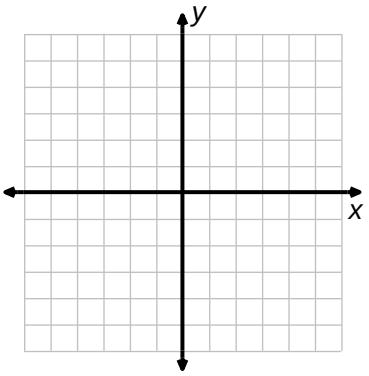
13. $(x - 3)^2 + (y + 2)^2 = 4$

13.



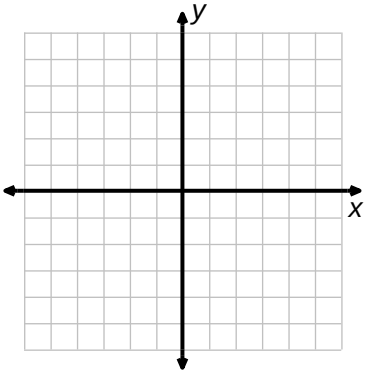
14. $y = -2x^2 + 4x - 1$

14.



15. $x + 3 = (y - 1)^2$

15.



Additional Exercises 10.1
Form II

Name _____

Date _____

Find the center and radius of the following circles.

1. $(x - 8)^2 + (y + 4)^2 = 25$ **1.** _____

2. $x^2 + y^2 + 2x - 6y + 1 = 0$ **2.** _____

3. $x^2 + y^2 + 4x - 12 = 0$ **3.** _____

Find the vertex of the following parabolas.

4. $y = (x + 12)^2 - 9$ **4.** _____

5. $y = x^2 + 14x + 38$ **5.** _____

6. $y = 2x^2 - 32x + 140$ **6.** _____

Write an equation of the circle with the given center and radius.

7. $(-13, 17)$; $r = 12$ **7.** _____

8. $(-8, -5); r = \sqrt{8}$ **8.** _____

9. $\left(\frac{5}{3}, -\frac{11}{3}\right); r = \frac{7}{3}$ **9.** _____

Write an equation of the parabola as a function of x or y with the given vertex and value for a. Determine whether the parabola opens upward, downward, to the left, or to the right.

10. $f(x)$ with vertex $(3, -6)$; $a = -8$ **10.** _____

11. $f(x)$ with vertex $(-8, -12)$; $a = -13$ **11.** _____

12. $g(y)$ with vertex $\left(\frac{7}{4}, \frac{17}{4}\right); a = 2$ **12.** _____

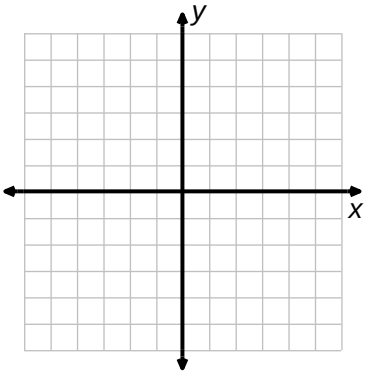
Additional Exercises 10.1 (*cont.*)

Name _____

Sketch the graph of each equation. If the graph is a parabola, find its vertex. If the graph is a circle, find its center and radius.

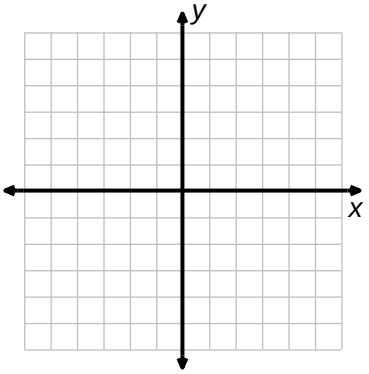
13. $y = (x - 3)^2 + 2$

13.



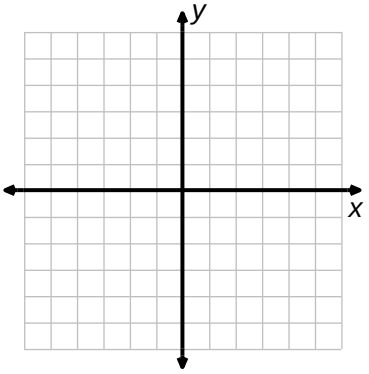
14. $x^2 + y^2 + 4x + 6y + 12 = 0$

14.



15. $y = 2x^2 - 2x + 3$

15.



Additional Exercises 10.1
Form III

Name _____

Date _____

Find the center and radius of the following circles.

1. $\left(x-\frac{5}{2}\right)^2+\left(y+\frac{1}{2}\right)^2=9$ **1.** _____

2. $x^2 + y^2 - 6x - 12y + 9 = 0$ **2.** _____

3. $x^2 + y^2 - 5x + 8y + 22 = 0$ **3.** _____

Find the vertex of the following parabolas.

4. $y=-2\left(x-\frac{3}{5}\right)^2+\frac{8}{5}$ **4.** _____

5. $y = 5x^2 - 40x + 73$ **5.** _____

6. $y = 3x^2 + 4x + 4$ **6.** _____

Write an equation of the circle with the given center and radius.

7. $(-7, 6)$; $r = 2.5$ **7.** _____

8. $(12,-15); r=\sqrt{17}$ **8.** _____

9. $\left(-\frac{9}{4},\frac{5}{4}\right); r=\frac{14}{5}$ **9.** _____

Write an equation of the parabola as a function of x or y with the given vertex and value for a. Determine whether the parabola opens upward, downward, to the left, or to the right.

10. $f(x)$ with vertex $\left(2,-\frac{7}{2}\right); a=-\frac{1}{3}$ **10.** _____

11. $g(y)$ with vertex $\left(\frac{1}{2},\frac{11}{2}\right); a=\frac{5}{3}$ **11.** _____

12. $f(x)$ with vertex $(-8, -25)$; $a = -0.4$ **12.** _____

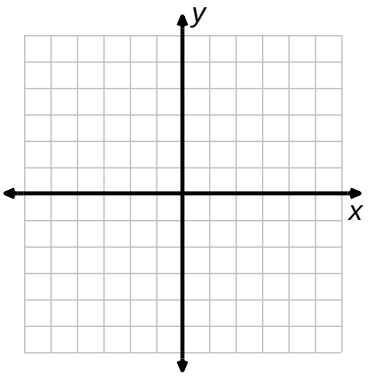
Additional Exercises 10.1 (*cont.*)

Name _____

Sketch the graph of each equation. If the graph is a parabola, find its vertex. If the graph is a circle, find its center and radius.

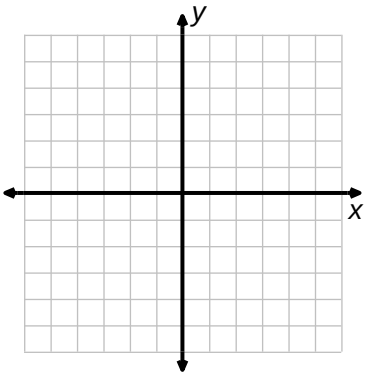
13. $x^2 + y^2 - 8x + 6y + 24 = 0$

13.



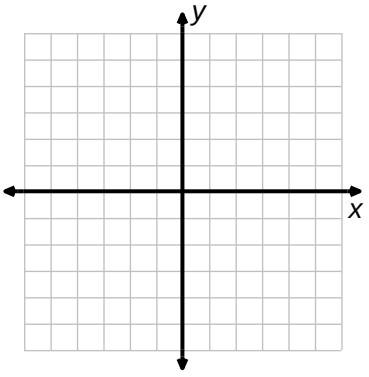
14. $y = -(x + 1)^2 + 3$

14.



15. $4x^2 + 4y^2 - 16x - 8y + 1 = 0$

15.



Additional Exercises 10.2
Form I

Name _____

Date _____

Identify whether each equation, when graphed, will be a parabola, circle, ellipse, or hyperbola.

1. $x^2 - y^2 = 9$ **1.** _____

2. $x^2 + y^2 = 12$ **2.** _____

3. $\frac{y^2}{9} - \frac{x^2}{27} = 1$ **3.** _____

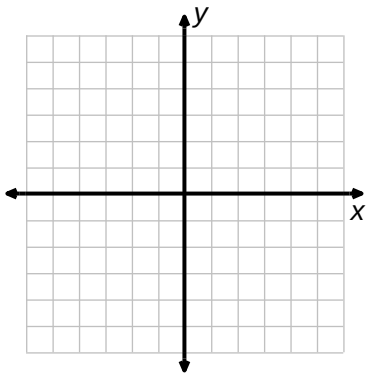
4. $\frac{x^2}{4} + \frac{y^2}{25} = 1$ **4.** _____

5. $4y = x^2 - 4$ **5.** _____

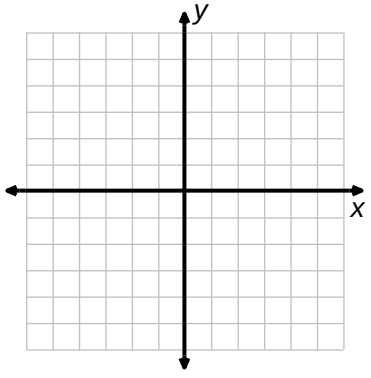
6. $x = -(y + 1)^2$ **6.** _____

Sketch the graph of each equation.

7. $\frac{x^2}{4} + \frac{y^2}{16} = 1$ **7.**



8. $9(x - 2)^2 + 4(y + 1)^2 = 36$ **8.**



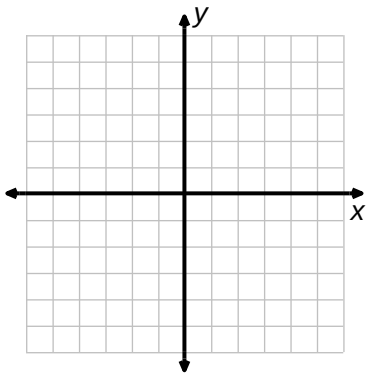
Additional Exercises 10.2 (*cont.*)

Name _____

Sketch the graph of each equation.

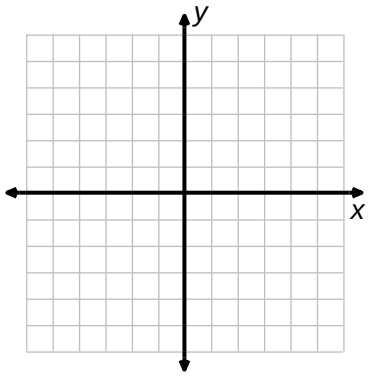
9. $y^2 - 9x^2 = 4$

9.



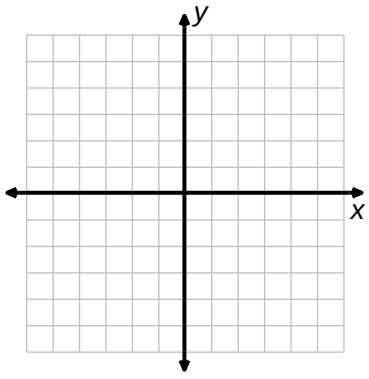
10. $x^2 - y^2 = 9$

10.



11. $x = 3(y + 1)^2$

11.



Additional Exercises 10.2
Form II

Name _____

Date _____

Identify whether each equation, when graphed, will be a parabola, circle, ellipse, or hyperbola.

1. $16x^2 - 4y^2 = 64$ **1.** _____

2. $(x + 4)^2 + (y - 8)^2 = 27$ **2.** _____

3. $\frac{y^2}{9} = 1 - \frac{x^2}{9}$ **3.** _____

4. $\frac{x^2}{4} + \frac{y^2}{4} = 1$ **4.** _____

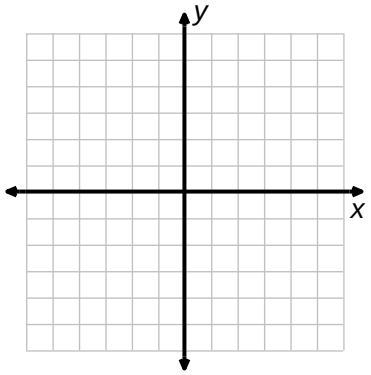
5. $-2y = x^2 + 4$ **5.** _____

6. $x^2 - (y + 3)^2 = 8$ **6.** _____

Sketch the graph of each equation.

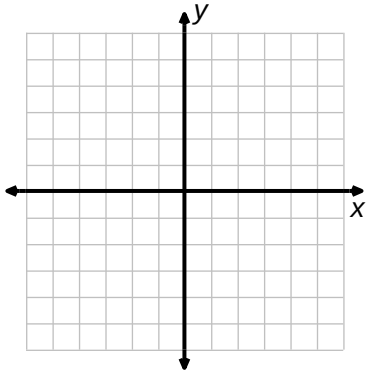
7. $\frac{x^2}{9} - \frac{y^2}{25} = 1$

7.



8. $\frac{(x+3)^2}{4} + (y-3)^2 = 1$

8.



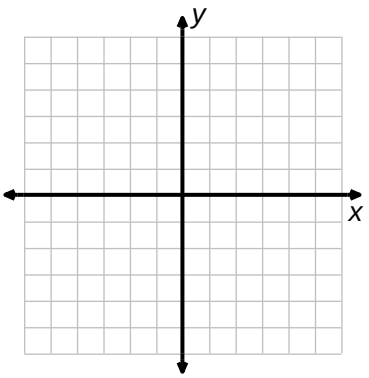
Additional Exercises 10.2 (*cont.*)

Name _____

Sketch the graph of each equation.

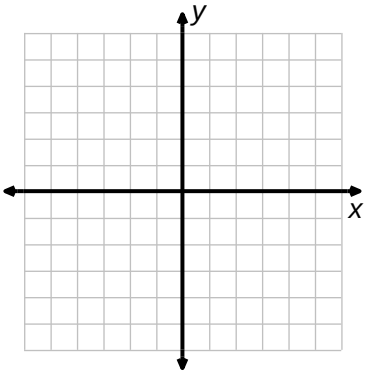
9. $y = -(x + 2)^2 + 3$

9.



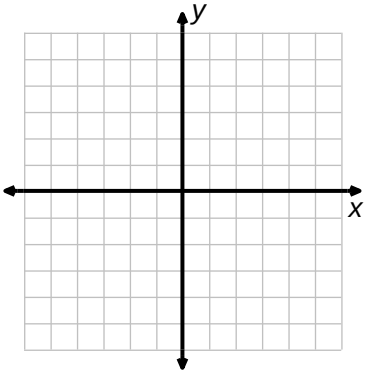
10. $5x^2 - y^2 = 25$

10.



11. $x^2 + y^2 + 2x + 4y - 4 = 0$

11.



Additional Exercises 10.2
Form III

Name _____

Date _____

Identify whether each equation, when graphed, will be a parabola, circle, ellipse, or hyperbola.

1. $5y^2 - 4x^2 = 80$ **1.** _____

2. $x^2 + y^2 - 4x + 6y - 3 = 0$ **2.** _____

3. $y = x^2 + 12x + 40$ **3.** _____

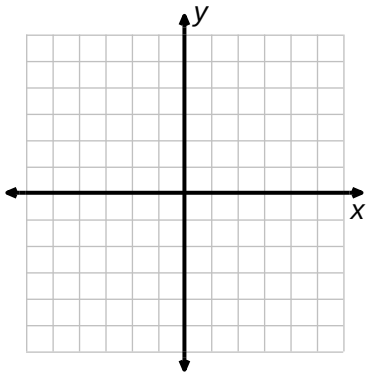
4. $5x^2 + 4y^2 = 20$ **4.** _____

5. $5x^2 = 25 + 3y^2$ **5.** _____

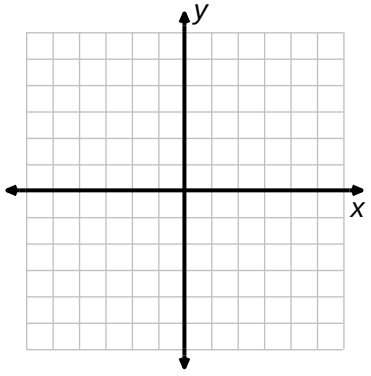
6. $\frac{x^2}{3} + \frac{y^2}{3} = 15$ **6.** _____

Sketch the graph of each equation.

7. $3y^2 - 2x^2 = 6$ **7.**



8. $y = x^2 - 2x + 1$ **8.**



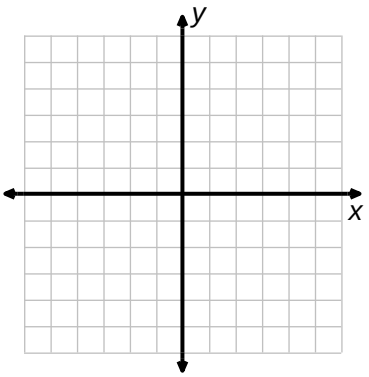
Additional Exercises 10.2 (*cont.*)

Name _____

Sketch the graph of each equation.

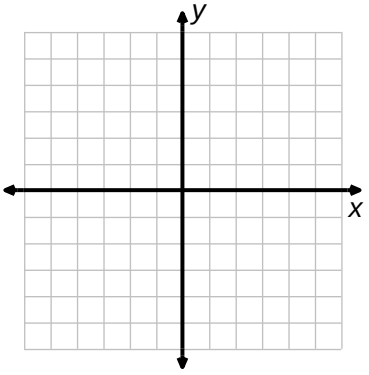
9. $y^2 + 3x^2 = 25$

9.



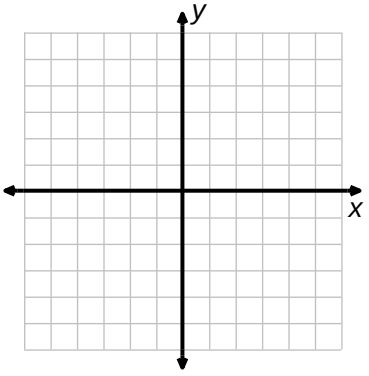
10. $x^2 + y^2 - 2x + 4y - 3 = 0$

10.



11. $32x^2 - 16 = 40y^2$

11.



Additional Exercises 10.3
Form I

Name _____

Date _____

Solve each nonlinear system of equations for real solutions.

1. $\begin{cases} x^2 + y^2 = 20 \\ y = -2x \end{cases}$ **1.** _____

2. $\begin{cases} 4x^2 - y^2 = 12 \\ 2x - y = 6 \end{cases}$ **2.** _____

3. $\begin{cases} 4x^2 - y^2 = 4 \\ 4x^2 + y^2 = 4 \end{cases}$ **3.** _____

4. $\begin{cases} x^2 + y^2 = 5 \\ y = 3x^2 - 1 \end{cases}$ **4.** _____

5. $\begin{cases} y = 2x^2 - 3 \\ y = 2x + 1 \end{cases}$ **5.** _____

6. $\begin{cases} x = y^2 - 5 \\ y = x - 1 \end{cases}$ **6.** _____

7. $\begin{cases} x^2 + y^2 = 13 \\ x^2 - y^2 = 5 \end{cases}$ **7.** _____

8. $\begin{cases} x^2 - y^2 = 12 \\ x^2 + 2y^2 = 24 \end{cases}$ **8.** _____

9. $\begin{cases} x^2 + y^2 = 10 \\ y = 2x \end{cases}$ **9.** _____

10. $\begin{cases} 4y^2 - x = 3 \\ 8y^2 - x^2 = -9 \end{cases}$ **10.** _____

11. $\begin{cases} 5x^2 - 4y^2 = 10 \\ x^2 + \dfrac{y^2}{4} = 2 \end{cases}$ **11.** _____

12. The difference of two numbers is 2 and the sum of their squares is 10. Find the numbers. **12.** _____

Additional Exercises 10.3
Form II

Name _____

Date _____

Solve each nonlinear system of equations for real solutions.

1. $\begin{cases} x^2 + y^2 = 8 \\ 4y = x^2 - 3 \end{cases}$ **1.** _____

2. $\begin{cases} x = 2y^2 - 1 \\ x = -2y^2 + 3 \end{cases}$ **2.** _____

3. $\begin{cases} 3x^2 - 2y^2 = -5 \\ 2x^2 - y^2 = -2 \end{cases}$ **3.** _____

4. $\begin{cases} x^2 + y^2 = 5 \\ y = 3x^2 - 1 \end{cases}$ **4.** _____

5. $\begin{cases} x^2 - 4y^2 = -7 \\ 3x^2 + y^2 = 31 \end{cases}$ **5.** _____

6. $\begin{cases} x^2 - y^2 = -4 \\ x^2 + y^2 = 11 \end{cases}$ **6.** _____

7. $\begin{cases} 2x^2 - y^2 = -6 \\ y = 2x^2 - 6 \end{cases}$ **7.** _____

8. $\begin{cases} 7x^2 - 3y^2 = -5 \\ 3x^2 + 5y^2 = 12 \end{cases}$ **8.** _____

9. $\begin{cases} x^2 - 2y^2 = -8 \\ 2x^2 + y^2 = 9 \end{cases}$ **9.** _____

10. $\begin{cases} x^2 - 8y^2 = -6 \\ 4y = x^2 - 6 \end{cases}$ **10.** _____

11. $\begin{cases} x^2 - y^2 + 2 = 0 \\ 3xy - 4 = 0 \end{cases}$ **11.** _____

12. The sum of two numbers is 7 and the difference of their squares is 21. Find the numbers. **12.** _____

Additional Exercises 10.3
Form III

Name _____

Date _____

Solve each nonlinear system of equations for real solutions.

1. $\begin{cases} x = y^2 - 2y + 6 \\ x = -y^2 + 4y + 6 \end{cases}$ **1.** _____

2. $\begin{cases} 2x^2 - 3y^2 = -3 \\ 3x^2 + y^2 = 23 \end{cases}$ **2.** _____

3. $\begin{cases} x^2 + y^2 - 2x + 4y - 20 = 0 \\ 2x + y = 5 \end{cases}$ **3.** _____

4. $\begin{cases} y^2 - 6x^2 = 57 \\ 3x - y = 15 \end{cases}$ **4.** _____

5. $\begin{cases} x^2 - 3y^2 = -1 \\ 2x^2 - 7y^2 = -5 \end{cases}$ **5.** _____

6. $\begin{cases} 2y^2 - x^2 = 4 \\ 11x + 4y = 14 \end{cases}$ **6.** _____

7. $\begin{cases} 4x^2 - y^2 = 24 \\ x^2 + 2y^2 = 16 \end{cases}$ **7.** _____

8. $\begin{cases} x^2 - 16y^2 = 16 \\ x - 24y = -23 \end{cases}$ **8.** _____

9. $\begin{cases} 2x^2 - 6y^2 + 3 = 0 \\ 4x^2 + 3y^2 - 4 = 0 \end{cases}$ **9.** _____

10. $\begin{cases} 2x^2 + y^2 = 2 \\ x^2 - 2y^2 = -8 \end{cases}$ **10.** _____

11. $\begin{cases} y = 4(x-3)^2 - 4 \\ y = 4x \end{cases}$ **11.** _____

12. The area of a garden is 45 square meters, while its perimeter is 28 meters. Find the dimensions of the garden. **12.** _____

Additional Exercises 10.4
Form I

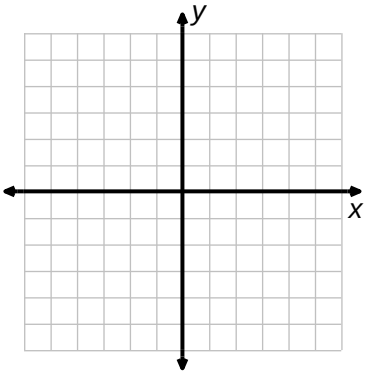
Name _____

Date _____

Graph each inequality.

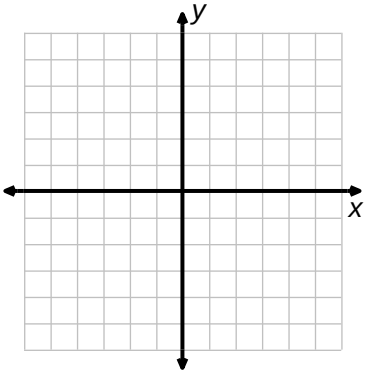
1. $x^2 + y^2 \leq 25$

1.



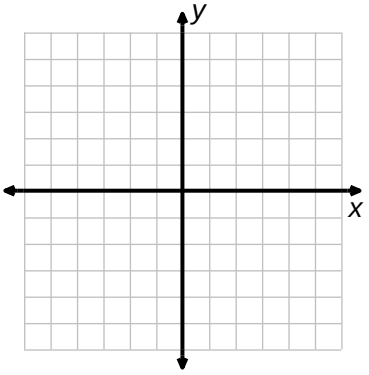
2. $(x - 2)^2 + (y + 1)^2 \leq 5$

2.



3. $x > y^2 + 2$

3.

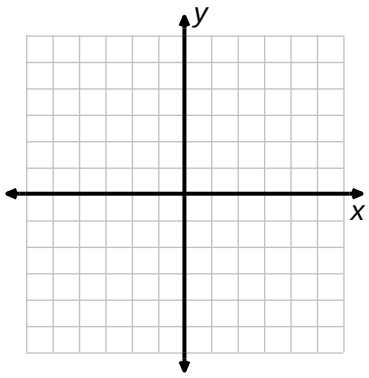


Additional Exercises 10.4 (*cont.*)

Name _____

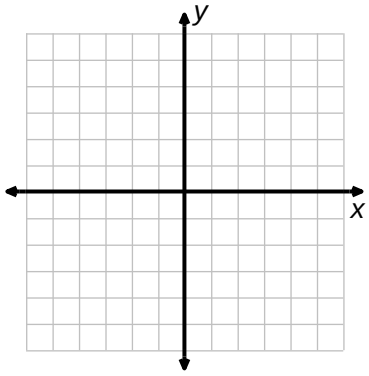
4. $x^2 - y^2 < 4$

4.



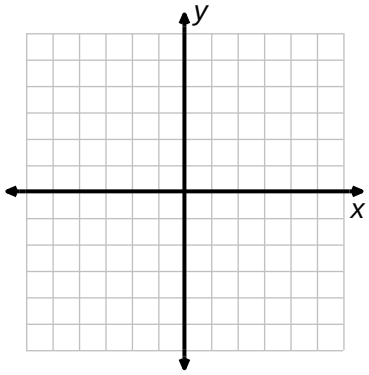
5. $\frac{x^2}{4} + \frac{y^2}{9} \leq 1$

5.



6. $y \geq (x - 2)^2 + 1$

6.



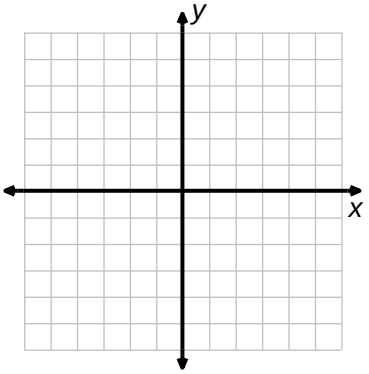
Additional Exercises 10.4 (*cont.*)

Name _____

Graph the solution of each system.

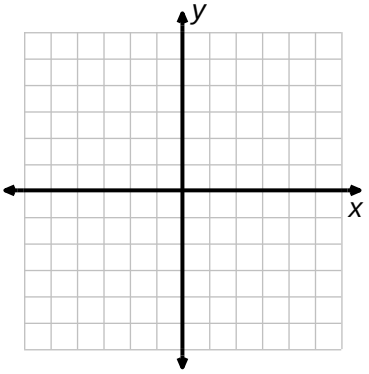
7. $\begin{cases} x - y > 1 \\ x^2 - 4y^2 < 8 \end{cases}$

7.



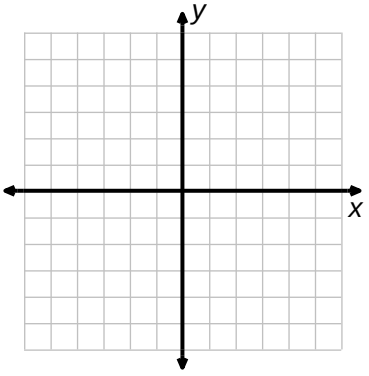
8. $\begin{cases} x^2 + 4y^2 \geq 16 \\ x^2 \leq y^2 + 4 \end{cases}$

8.



9. $\begin{cases} y \leq x + 3 \\ y \geq x^2 \end{cases}$

9.

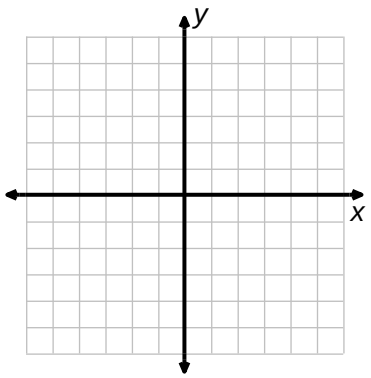


Additional Exercises 10.4 (*cont.*)

Name _____

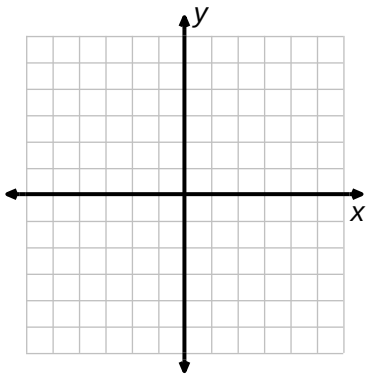
10. $\begin{cases} x^2 \geq -y^2 \\ x^2 + y^2 \leq 9 \end{cases}$

10.



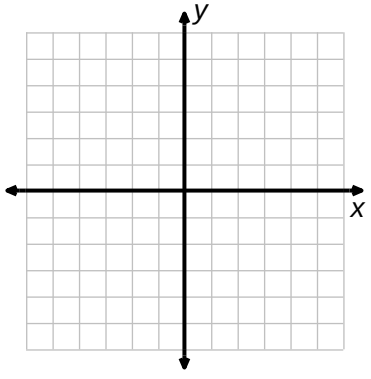
11. $\begin{cases} x^2 + 4y^2 \leq 4 \\ (x-1)^2 + (y+1)^2 \geq 1 \end{cases}$

11.



12. $\begin{cases} x - y^2 \leq 2 \\ x - 2y + 2 > 0 \end{cases}$

12.



Additional Exercises 10.4
Form II

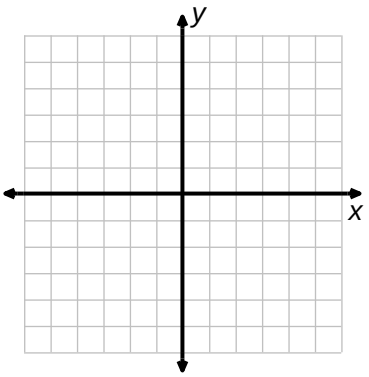
Name _____

Date _____

Graph each inequality.

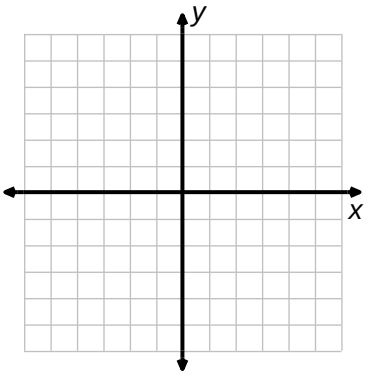
1. $\frac{x^2}{4} + \frac{y^2}{25} \leq 1$

1.



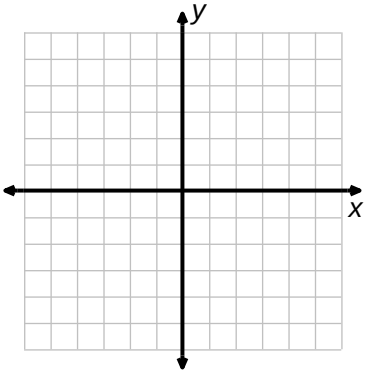
2. $y \geq x^2 - 6x + 10$

2.



3. $4x^2 - y^2 \geq 16$

3.

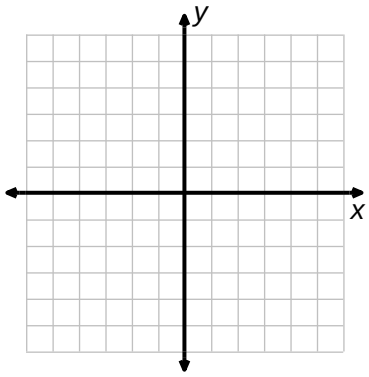


Additional Exercises 10.4 (*cont.*)

Name _____

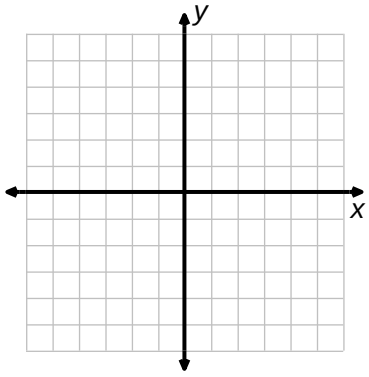
4. $x^2 + y^2 + 2x - 6y < -6$

4.



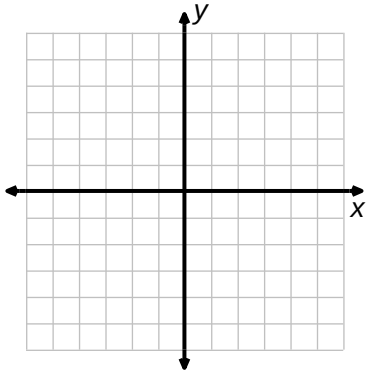
5. $x^2 + y^2 - 1 \geq 4y - 4x$

5.



6. $y \geq -x^2 - 4x - 1$

6.



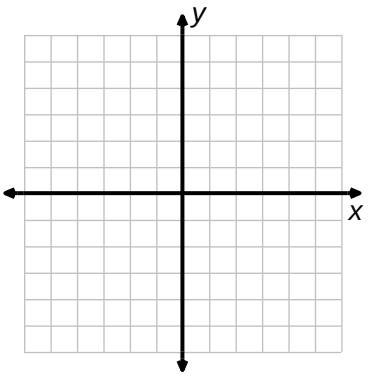
Additional Exercises 10.4 (*cont.*)

Name _____

Graph the solution of each system.

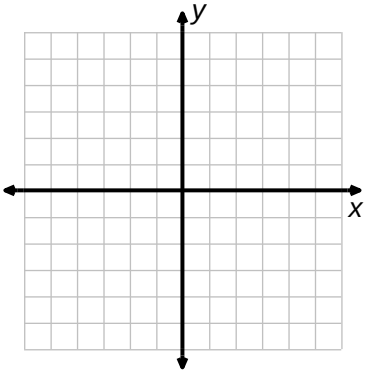
7. $\begin{cases} 3x^2 + 16y < 48 \\ 4x + 7y \geq 5 \end{cases}$

7.



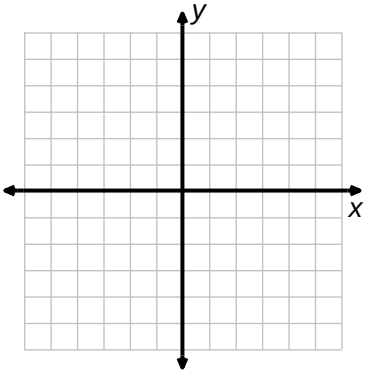
8. $\begin{cases} 4(x-1)^2 + y^2 \geq 16 \\ (x-1)^2 + 4y^2 \leq 16 \end{cases}$

8.



9. $\begin{cases} x^2 + y^2 + 2x \leq 4 + 4y \\ x - 2y + 2 < 0 \end{cases}$

9.

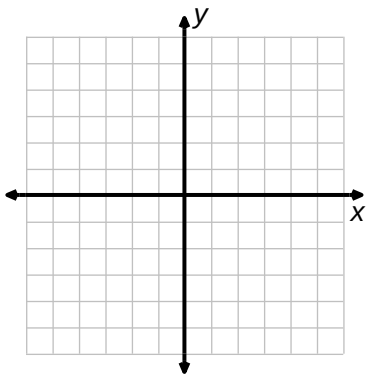


Additional Exercises 10.4 (*cont.*)

Name _____

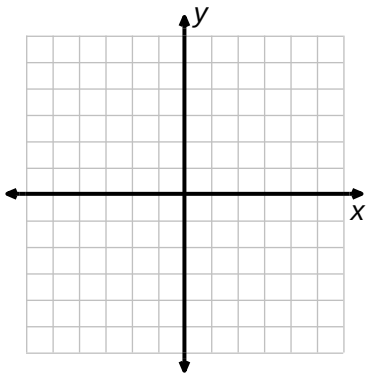
10. $\begin{cases} x^2 + y^2 + 2x - 2y \le 7 \\ 5x + 9y \le 2 \end{cases}$

10.



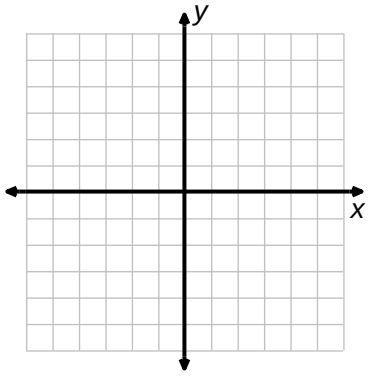
11. $\begin{cases} x^2 + 5y \le 5 \\ x^2 + 2y^2 < 8 \end{cases}$

11.



12. $\begin{cases} y^2 - 4x^2 < 4 \\ y < 2x - 1 \end{cases}$

12.



Additional Exercises 10.4
Form III

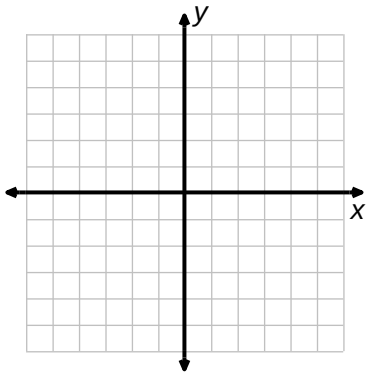
Name _____

Date _____

Graph each inequality.

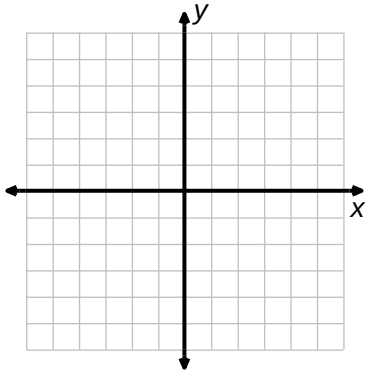
1. $x^2 - 3(y - 2)^2 \geq 9$

1.



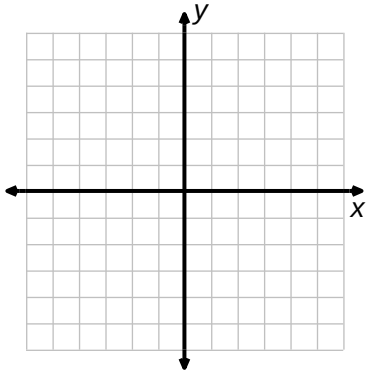
2. $x < y^2 - 6y + 7$

2.



3. $4(y + 1)^2 - 3(x - 2)^2 \leq 9$

3.

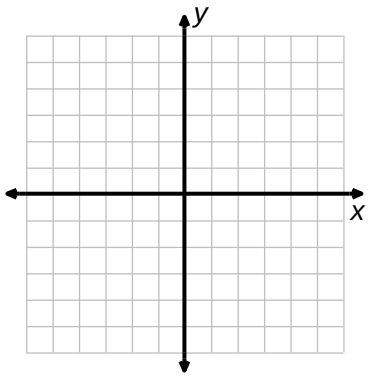


Additional Exercises 10.4 (*cont.*)

Name _____

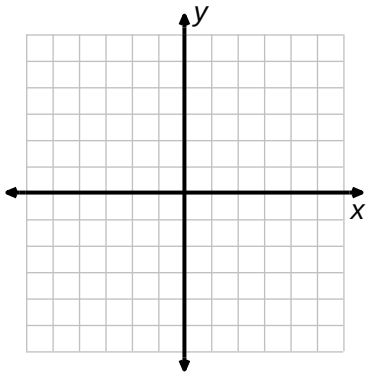
4. $2(x + 1)^2 \leq 8 - y^2$

4.



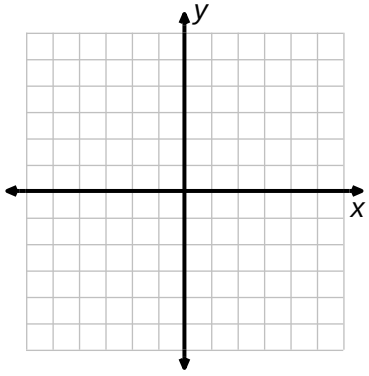
5. $y > x^2 + 6x + 6$

5.



6. $9x^2 - 36x + 4y^2 - 16y > -16$

6.



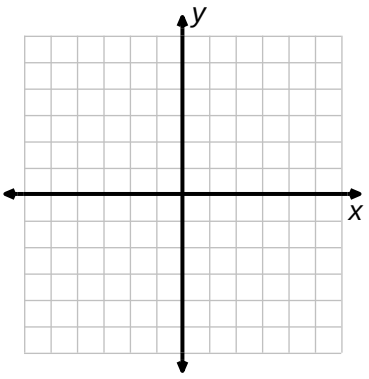
Additional Exercises 10.4 (*cont.*)

Name _____

Graph the solution of each system.

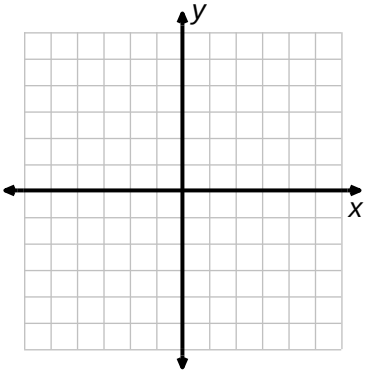
7. $\begin{cases} 4(x-1)^2 + 2y^2 \geq 4 \\ \frac{1}{2}x + y \geq 4 \end{cases}$

7.



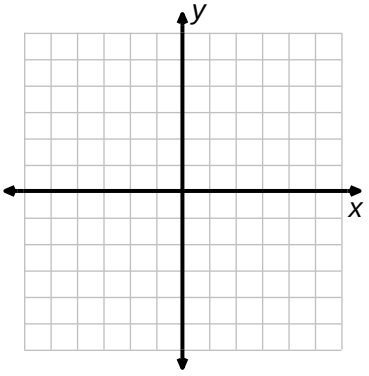
8. $\begin{cases} 2x^2 + 4y^2 \leq 8 \\ 3y^2 > x^2 + 9 \end{cases}$

8.



9. $\begin{cases} x^2 \leq 4 - 6y \\ 4x^2 + y^2 > 9 \end{cases}$

9.

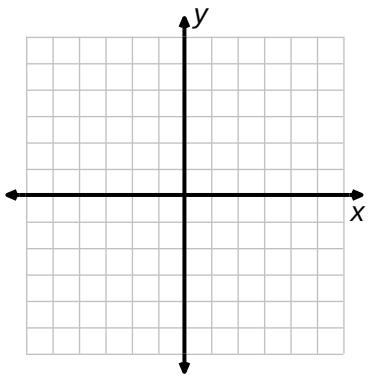


Additional Exercises 10.4 (*cont.*)

Name _____

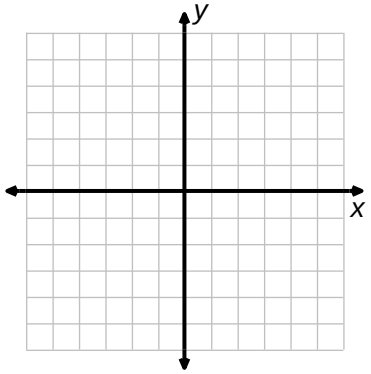
10. $\begin{cases} x - y^2 \le 2 \\ 4x^2 - 2y^2 > 9 \end{cases}$

10.



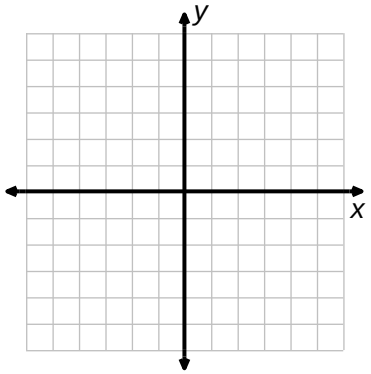
11. $\begin{cases} 2x - y \le 3 \\ (x-3)^2 - (y-2)^2 > 4 \\ 3 - x > y \end{cases}$

11.



12. $\begin{cases} y \le 2(x-2)^2 \\ 3x^2 + (y-2)^2 \le 3 \\ y > \frac{3}{2}x + 4 \end{cases}$

12.

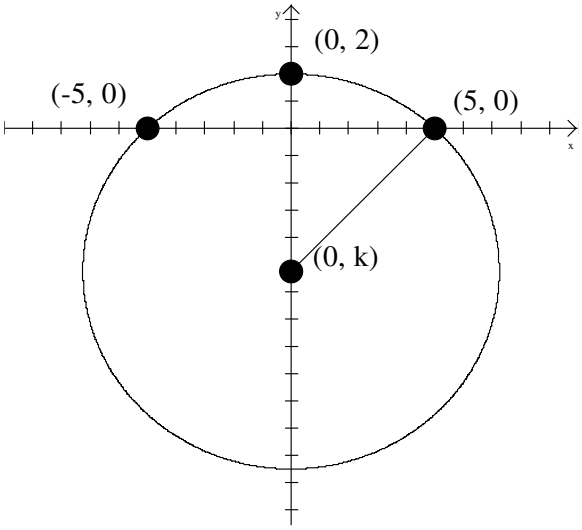


Name: **Date:**
Instructor: **Section:**

Section 10.3 Solving Nonlinear Systems of Equations

Objective: Solve problems that can be modeled by nonlinear systems of equations.
Suggested Format: Small Group Format
Time: 20 minutes

Many modern homes today have windows with arches on the top. On one particular home-site, a contractor needs to cut a hole into a wall to accommodate a window with a circular arch that is 10 ft wide and 2 ft high. In order to draw the arch, the contractor will use a piece of string with a pencil attached to the end to act as a compass. He will need to find where to anchor the string (for the center) and what the radius should be. Study the picture below.



Use the equation $(x-h)^2+(y-k)^2=r^2$ and the points (5, 0) on the circle to write an equation. (HINT: If (h, k) is the center, what will h = ?) Plug in the numbers 5 and 0 for x and y and simplify the equation. Do the same thing for the point (0, 2). Use your two equation to solve for k and r.

1. Where is the string of the compass anchored (the center)? How long is the string (the radius)?

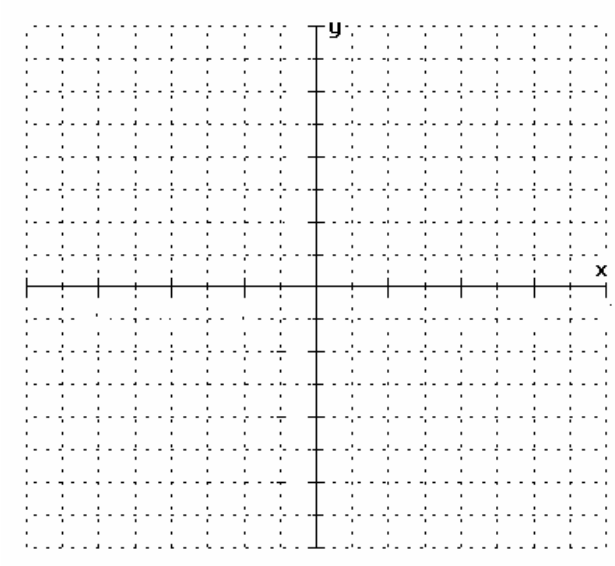
2. Write the equation of the circle using the information you found.

Name: **Date:**
Instructor: **Section:**

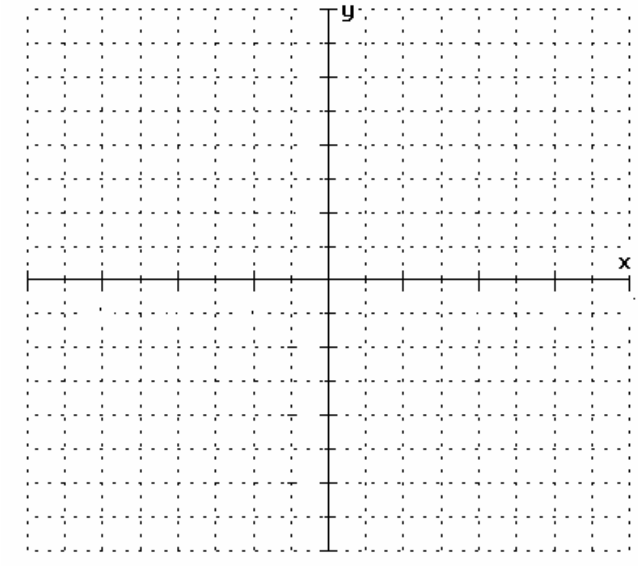
Section 10.4 Nonlinear Inequalities and Systems of Inequalities

Objective: Graph solution sets to nonlinear systems of inequalities.
Suggested Format: Small Group Format
Time: 10 minutes

1. Graph the solution set. $\begin{cases} x^2+6x+5 < y \\ (x-3)^2+(y+2)^2 < 9 \end{cases}$



2. Write a system of nonlinear inequalities that has no solution. (HINT: Draw a graph first.)



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Chapter 10 Test Form A

Identify whether each equation, when graphed, will be a parabola, circle, ellipse, or hyperbola.

1. $\frac{(x-2)^2}{4}+\frac{(y-3)^2}{9}=1$ **1.** _____

2. $x=(y-3)^2+4$ **2.** _____

3. $\frac{(y+7)^2}{4}=\frac{(x+6)^2}{11}-12$ **3.** _____

4. $2(y-1)^2=7-x$ **4.** _____

5. $2(x+2)^2=-2y^2-6$ **5.** _____

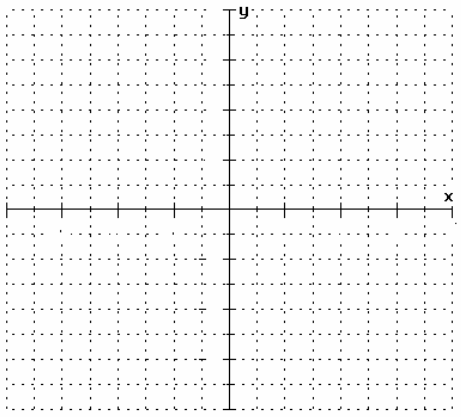
6. Give the center and radius of the circle. $x^2+y^2=14x+6y+23$ **6.** _____

7. Give the vertex of the parabola and indicate whether it opens upwards, downwards, to the left, or to the right. $2x^2+16x+y+32=0$ **7.** _____

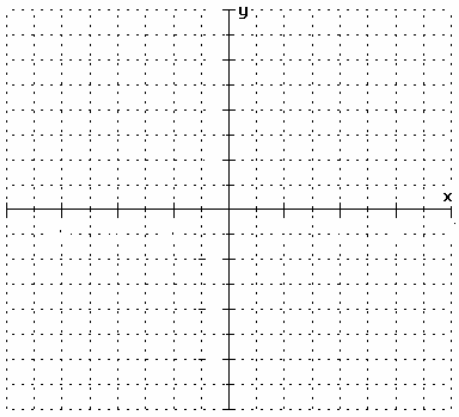
8. Give the equation in standard form of the circle with center (−2, 5) and radius 6. **8.** _____

Graph each equation.

9. $x^2+y^2+4x=12$

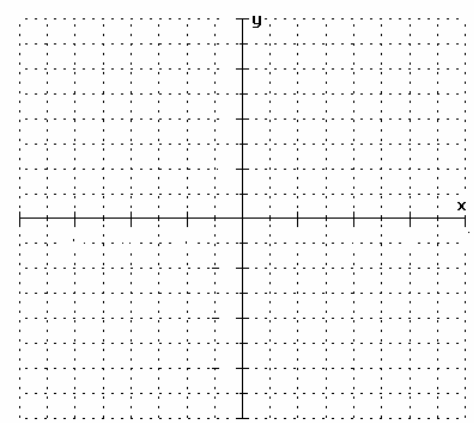


10. $x^2-9y^2+9=0$

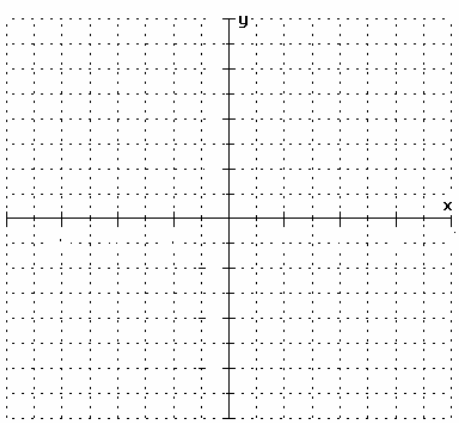


Chapter 10 Test Form A *cont'd*

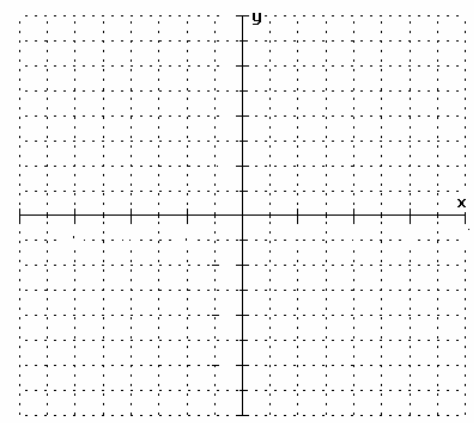
11. $\frac{(x+3)^2}{4}+(y-2)^2=1$



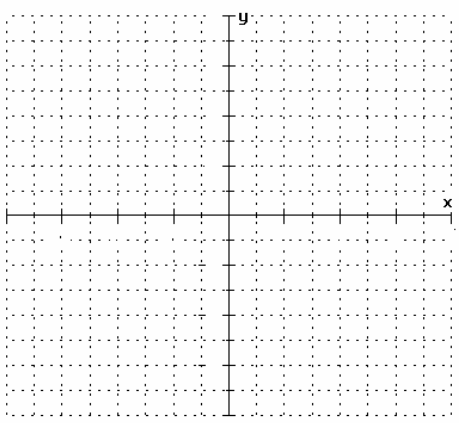
12. $x=-(y-2)^2+3$



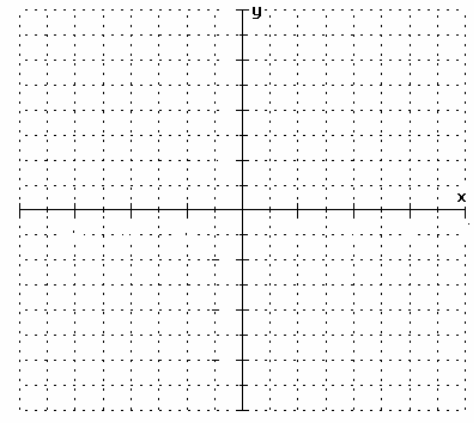
13. $y-3=2x^2-4x$



14. $4(x+1)^2-(y-2)^2=16$



15. $4x^2+9y^2=36$



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Chapter 10 Test Form A *cont'd*

Solve each system of equations.

16. $\begin{cases} x^2 + y^2 = 36 \\ x^2 + 2y^2 = 62 \end{cases}$ **16.** _____

17. $\begin{cases} 4x^2 - y^2 = 4 \\ 4x^2 + y^2 = 4 \end{cases}$ **17.** _____

18. $\begin{cases} x^2 + y^2 = 4 \\ x^2 + 3y = 0 \end{cases}$ **18.** _____

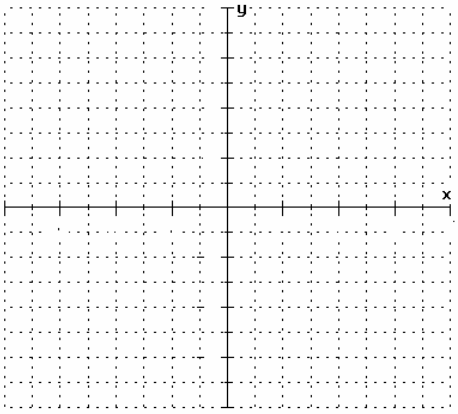
19. A rectangular frame is to be designed so that its perimeter is 34 inches and its diagonal is 13 inches. Find the dimensions of the frame. **19.** _____

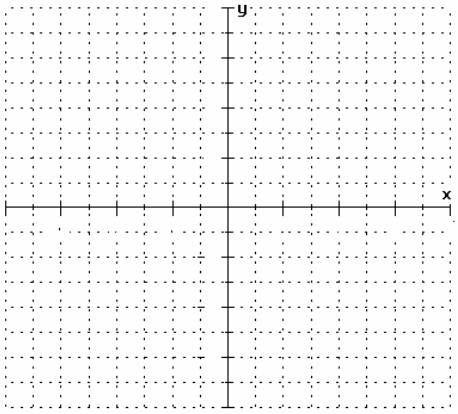
20. What is the maximum number of real solutions for a system of equations for which the graphs are a circle and a hyperbola? **20.** _____

Graph each inequality.

21. $x^2 - 4y^2 < 4$

22. $\frac{x^2}{16} + \frac{y^2}{9} > 1$





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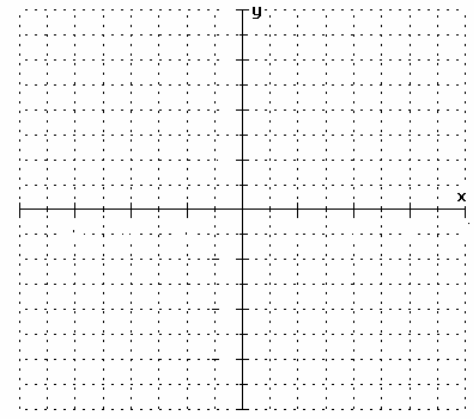
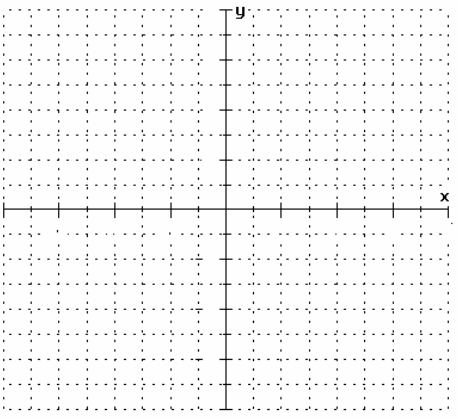
Chapter 10 Test Form A *cont'd*

Graph the solution of each system.

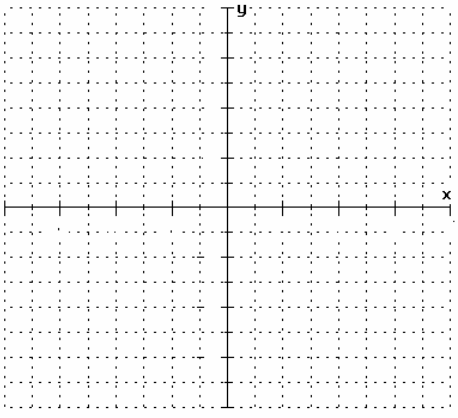
23. $\begin{cases} x^2+4x-2y\ge 0 \\ x^2+y^2+4x+4y<1 \end{cases}$



24. $\begin{cases} x\le (y+2)^2-1 \\ (x-2)^2+(y+2)^2\ge 9 \end{cases}$



25. $\begin{cases} \dfrac{x^2}{9}+y^2<1 \\ x^2-\dfrac{y^2}{4}\ge 1 \end{cases}$



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Chapter 10 Test Form B

Identify whether each equation, when graphed, will be a parabola, circle, ellipse, or hyperbola.

1. $y+4=(x-3)^2$ **1.** _____

2. $x^2+y^2+6x+4y+12=0$ **2.** _____

3. $4y^2=30x+4x^2$ **3.** _____

4. $24x-x^2=y^2$ **4.** _____

5. $\frac{(x-2)^2}{4}+\frac{(y+6)^2}{3}=12$ **5.** _____

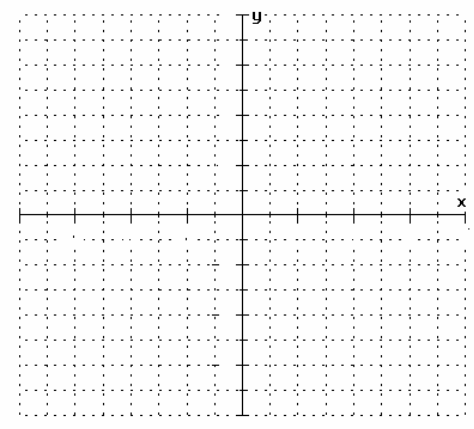
6. Give the center and radius of the circle. $x^2+y^2+16x-26y+197=0$ **6.** _____

7. Give the vertex of the parabola and indicate whether it opens upwards, downwards, to the left, or to the right. $2y^2+4y+3-x=0$ **7.** _____

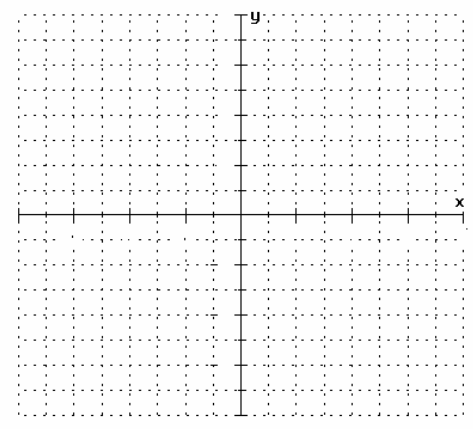
8. Give the equation in standard form of the circle with center (–2, –2) and radius 5. **8.** _____

Graph each equation.

9. $x^2+y^2-6x-4y+9=0$



10. $4y^2-x^2=8$



Name: **Date:**
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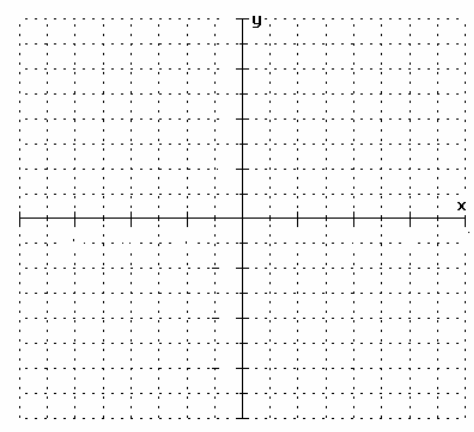
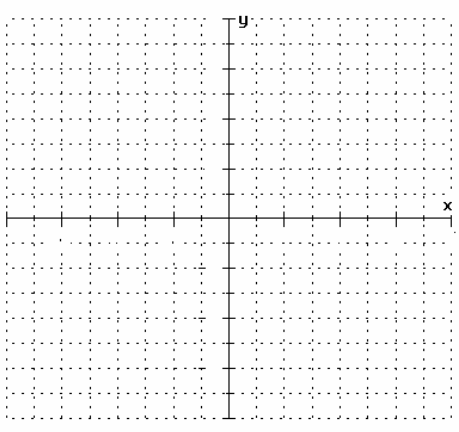
Chapter 10 Test Form B *cont'd*

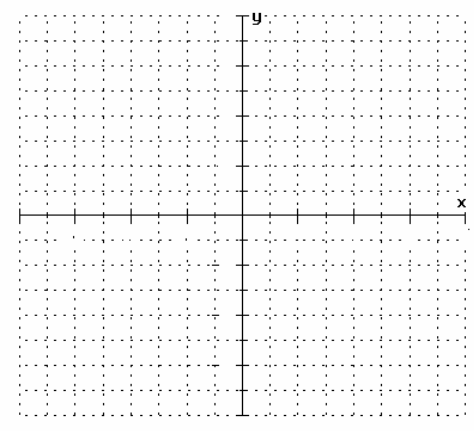
11. $\frac{(x-1)^2}{9}+\frac{(y-2)^2}{4}=1$



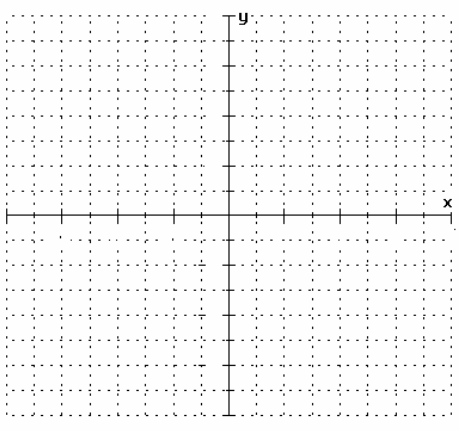
12. $x=-(y+4)^2+4$



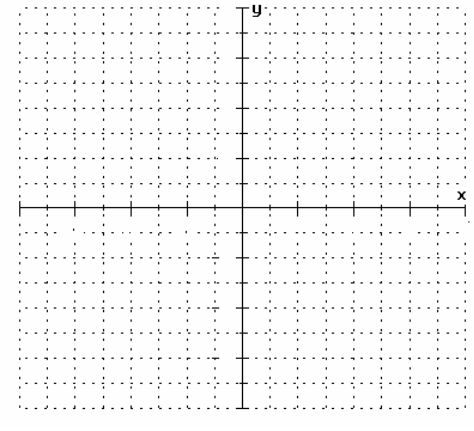
13. $y-4=-2x^2+4x$



14. $9x^2-4(y-2)^2=36$



15. $x^2+4y^2=16$



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Chapter 10 Test Form B *cont'd*

Solve each system of equations.

16. $\begin{cases} 3x^2 - 2y^2 = -5 \\ 2x^2 - y^2 = -2 \end{cases}$ **16.** _____

18. $\begin{cases} x^2 + y^2 = 17 \\ x + 4y = 0 \end{cases}$ **17.** _____

18. $\begin{cases} y^2 + 4 = x^2 \\ 2x^2 + 3y^2 = 6 \end{cases}$ **18.** _____

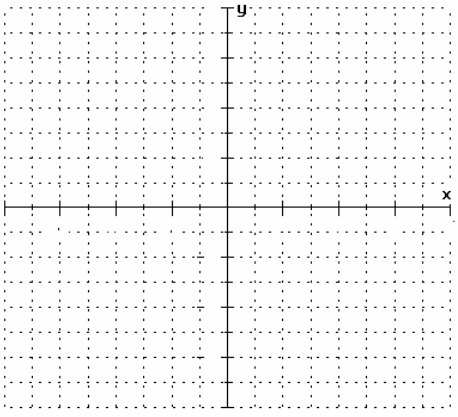
19. The product of two positive numbers is 36. The sum of the squares of the two numbers is 328. Find the two numbers. **19.** _____

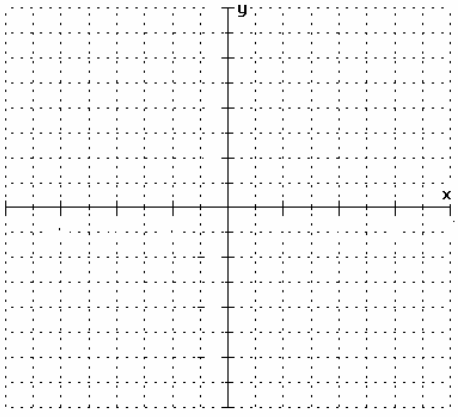
20. What is the maximum number of real solutions for a system of equations for which the graphs are a line and a hyperbola? **20.** _____

Graph each inequality.

21. $x > (y-3)^2 + 2$

22. $\frac{(x-1)^2}{4} + \frac{(y+2)^2}{9} > 1$





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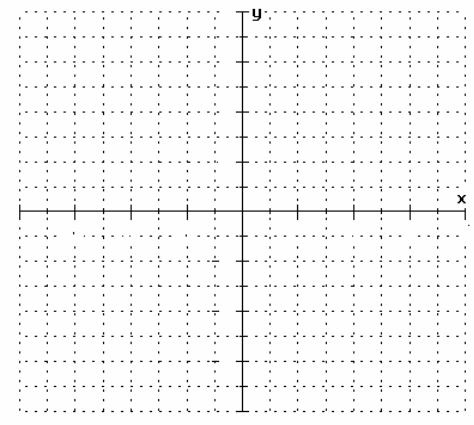
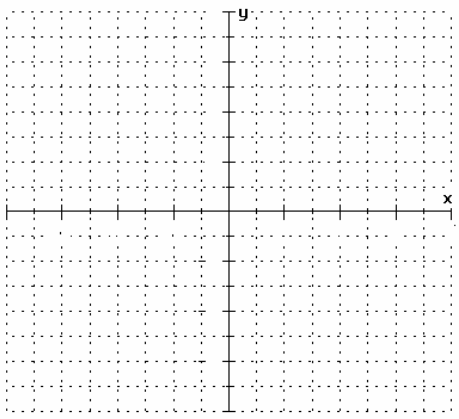
Chapter 10 Test Form B *cont'd*

Graph the solution of each system.

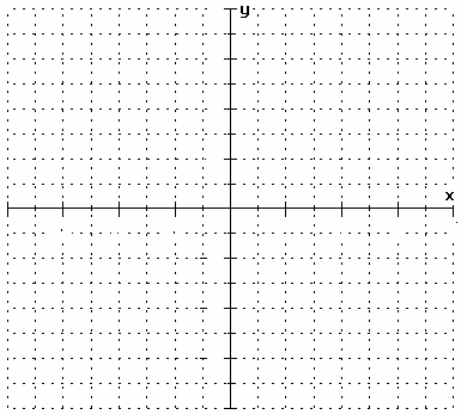
23. $\begin{cases} x^2 + y^2 + 2x - 2y \le 7 \\ 5x + 9y > 2 \end{cases}$



24. $\begin{cases} x^2 + y^2 < 9 \\ 2x + 3y > 6 \end{cases}$



25. $\begin{cases} x \ge y^2 - 6y + 5 \\ x^2 + y^2 \le 4 \end{cases}$



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Chapter 10 Test Form C

Identify whether each equation, when graphed, will be a parabola, circle, ellipse, or hyperbola.

1. $(x-3)^2+4(y-4)^2=16$ **1.** _____

2. $x^2+y^2+2x+4y+12=0$ **2.** _____

3. $5x^2-2=5y^2-3y+1$ **3.** _____

4. $(x-2)^2-y=3$ **4.** _____

5. $\frac{x^2}{4}+\frac{(y-1)^2}{4}=2$ **5.** _____

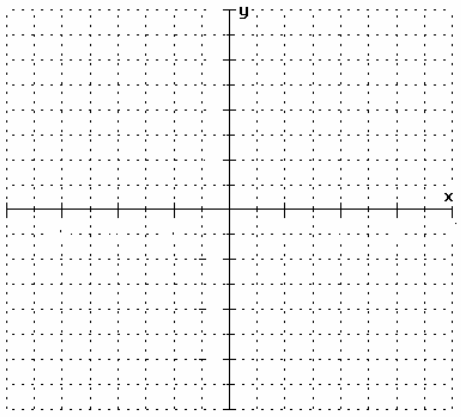
6. Give the center and radius of the circle. $16x^2+16y^2+8x-10=0$ **6.** _____

7. Give the vertex of the parabola and indicate whether it opens upwards, downwards, to the left, or to the right. $x=-2(y+5)^2-8$ **7.** _____

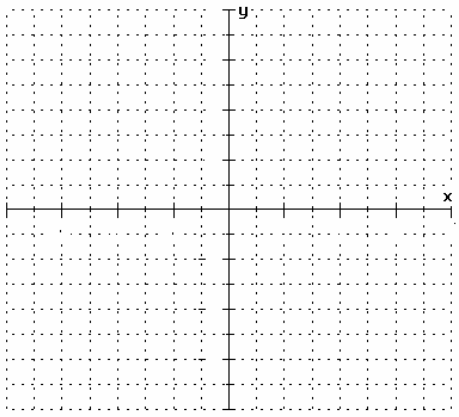
8. Give the equation in standard form of the circle with center (–4, 3) and radius $\sqrt{3}$. **8.** _____

Graph each equation.

9. $x^2+y^2+8x-6y+24=0$



10. $y^2-x^2=9$



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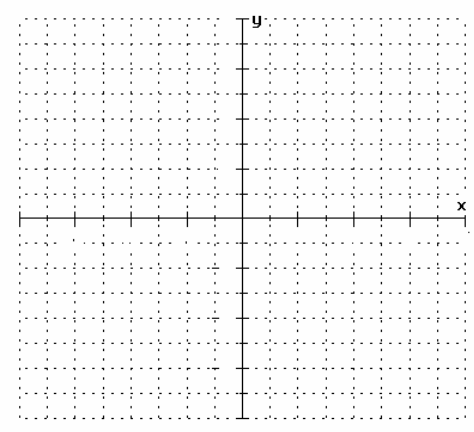
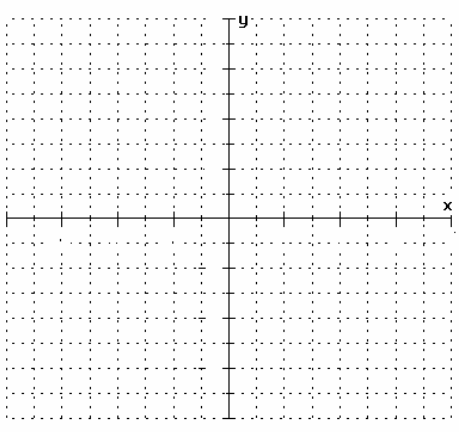
Chapter 10 Test Form C *cont'd*

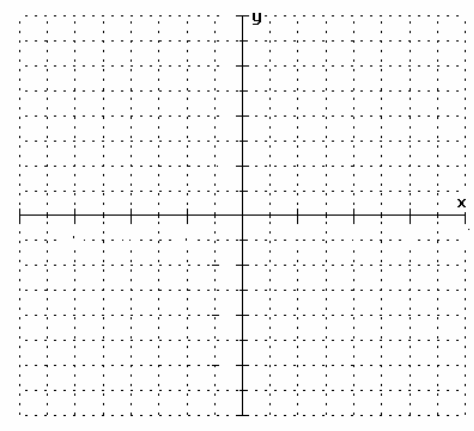
11. $\frac{(x-2)^2}{4}+\frac{(y-3)^2}{9}=1$



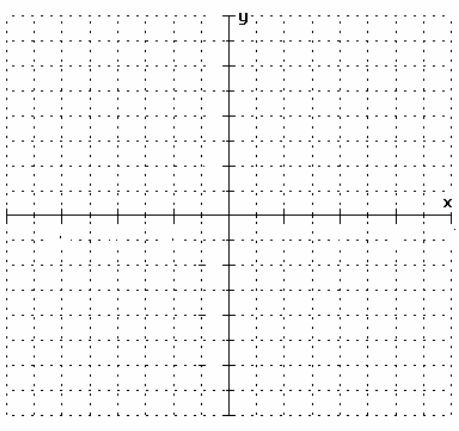
12. $x=-(y-3)^2+4$



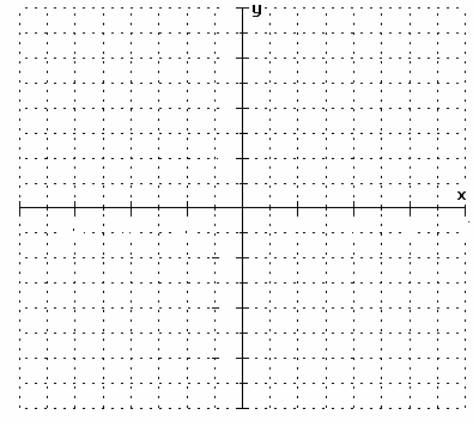
13. $3-y=-x^2+4x+4$



14. $(y+2)^2-(x-3)^2=1$



15. $x^2+4y^2-2x-3=0$



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Chapter 10 Test Form C *cont'd*

Solve each system of equations.

16. $\begin{cases} 5x^2 + 2y^2 = 12 \\ 5x^2 + 4y = 6 \end{cases}$ **16.** _____

19. $\begin{cases} x^2 + y^2 = 4 \\ y^2 - x = 4 \end{cases}$ **17.** _____

18. $\begin{cases} y = x^2 - 4 \\ y = 6x - 13 \end{cases}$ **18.** _____

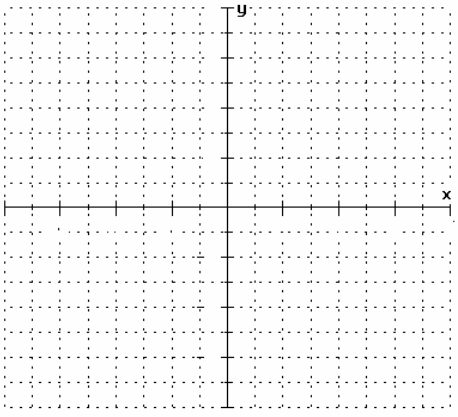
19. What is the maximum number of real solutions for a system of equations for which the graphs are an ellipse circle and a circle? **19.** _____

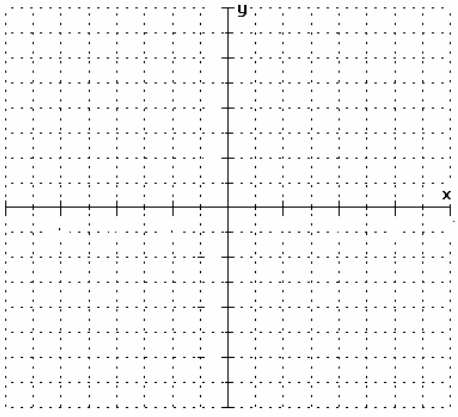
20. The sum of two numbers is 20. The difference between the squares of the two numbers is 120. Find the two numbers. **20.** _____

Graph each inequality.

21. $x^2 + y^2 - 4x - 2y \le 4$

22. $\frac{(x+1)^2}{4} + \frac{(y+2)^2}{9} < 1$





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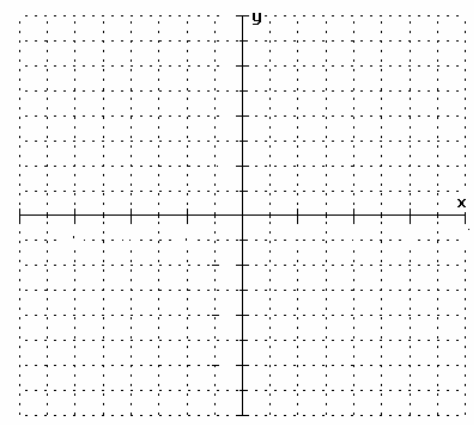
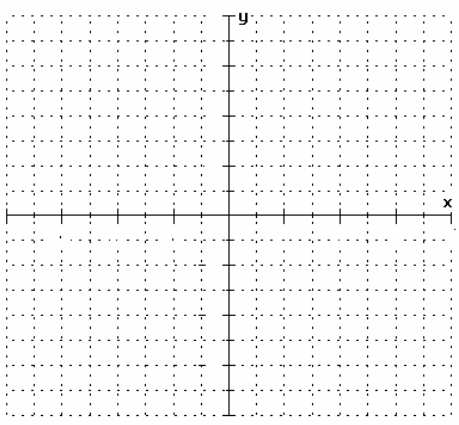
Chapter 10 Test Form C *cont'd*

Graph the solution of each system.

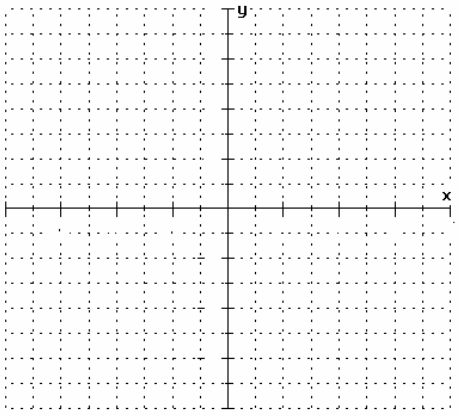
23. $\begin{cases} x - y > 1 \\ x^2 - 4y < 8 \end{cases}$

24. $\begin{cases} y > -(x-3)^2 + 4 \\ \frac{x^2}{9} + \frac{y^2}{4} \le 1 \end{cases}$





25. $\begin{cases} x^2 + (y-2)^2 \le 4 \\ 9y^2 \le 9 + x^2 \end{cases}$



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Chapter 10 Test Form D

Circle the correct answer.

1. Find the center and radius of the circle. $x^2 + y^2 - 12x + 10y + 59 = 0$

a. (6, –5); 2 **b.** (–6, 5); $\sqrt{2}$ **c.** (6, –5); $\sqrt{2}$ **d.** (–6, 5); 2

2. Find the center and radius of the circle. $4x^2 + 4y^2 + 24x - 32y + 99 = 0$

a. (–3, 4); $\frac{1}{2}$ **b.** (3, –4); $\frac{1}{2}$ **c.** (–3, 4); $\frac{1}{4}$ **d.** (3, –4); $\frac{1}{4}$

Use the following equation for problems 3 and 4. $y = -x^2 - 4x + 5$

3. Find the vertex of the parabola.

a. (2, –7) **b.** (–2, 17) **c.** (–2, 9) **d.** (2, 1)

4. The graph of the parabola would open

a. upward **b.** downward **c.** right **d.** left

Identify the graph of each function.

5. $2x^2 + y + 4x - 1 = 0$

a. parabola opening upward
b. parabola opening downward
c. parabola opening to the right
d. parabola opening to the left

6. $x + 5y^2 = 20y - 27$

a. parabola opening upward
b. parabola opening downward
c. parabola opening to the right
d. hyperbola

7. $(x-3)^2 - y^2 = 36$

a. circle centered at (3, 0)
b. parabola opening upward
c. ellipse
d. circle centered at (–3, 0)

8. $3x^2 + y^2 - 3x + y = 0$

a. parabola **b.** circle **c.** ellipse **d.** hyperbola

Chapter 10 Test Form D *cont'd*

9. $y = -2x^2 - 5x + 2$

a. parabola **b.** circle **c.** ellipse **d.** hyperbola

10. $3(x+3)^2 + 3y^2 = 10y$

a. parabola **b.** circle **c.** ellipse **d.** hyperbola

11. $x^2 + 6x + y^2 + 4y = 0$

a. parabola **b.** circle **c.** ellipse **d.** hyperbola

12. $x - 6 = y^2 + 2y$

a. parabola **b.** circle **c.** ellipse **d.** hyperbola

13. $3(x+3)^2 + 3(y-2)^2 = 8$

a. parabola **b.** circle **c.** ellipse **d.** hyperbola

14. $x^2 = y^2 + 2y - 4$

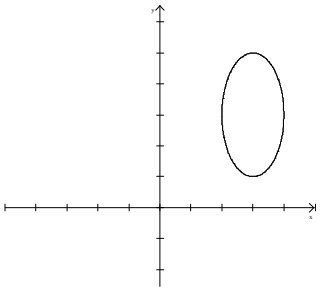
a. parabola **b.** circle **c.** ellipse **d.** hyperbola

Chapter 10 Test Form D *cont'd*

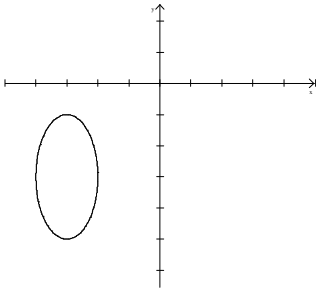
Match the graph to the equation.

15. $4x^2 + y^2 + 24x - 6y + 41 = 0$

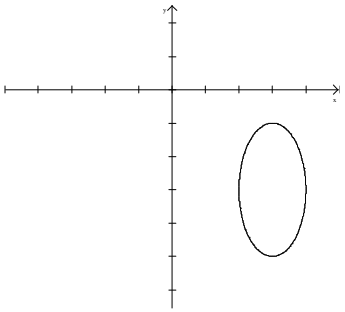
a.



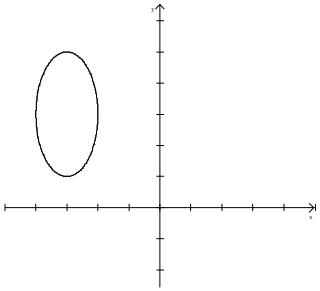
b.



c.

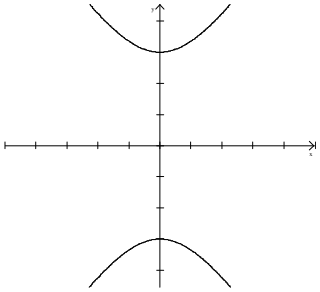


d.

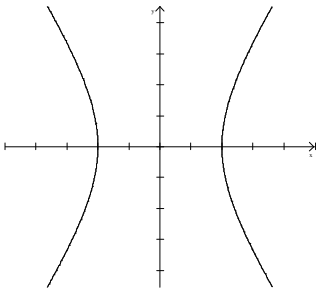


16. $9x^2 - 4y^2 = 36$

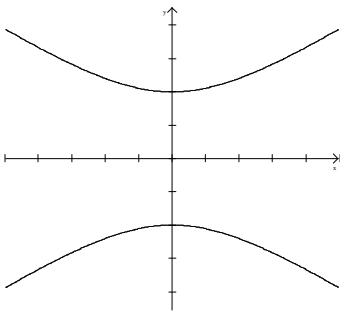
a.



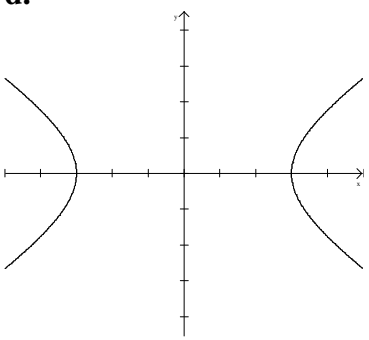
b.



c.



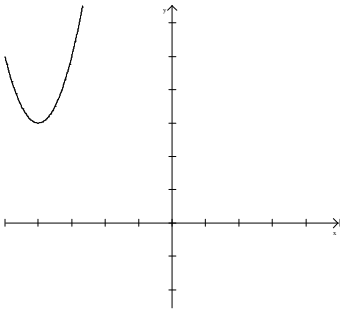
d.



Chapter 10 Test Form D *cont'd*

17. $x = 2(y-3)^2 - 4$

a.



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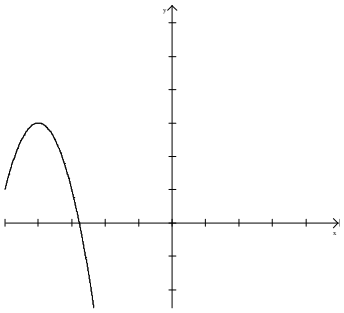
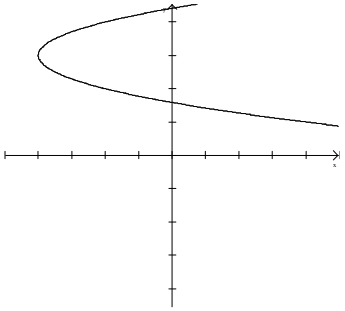
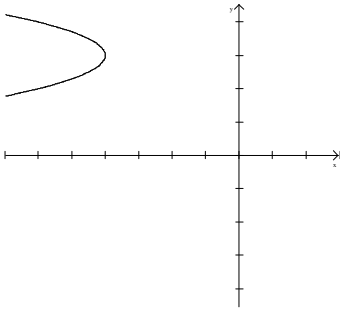


c.

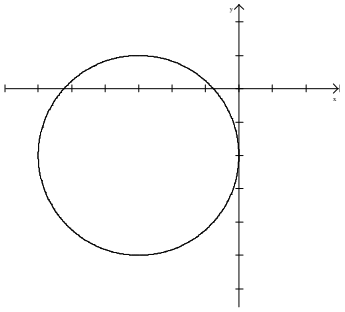


d.

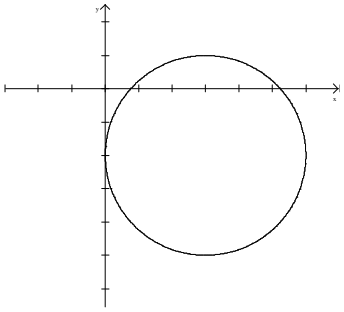


18. $x^2 + y^2 - 6x + 4y + 4 = 0$

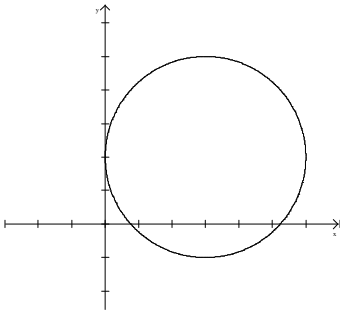
a.



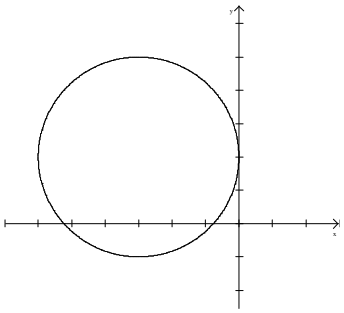
b.



c.



d.



Chapter 10 Test Form D *cont'd*

Solve each system on equations.

19. $\begin{cases} x^2 + (y-2)^2 = 65 \\ 4x - 7y = -14 \end{cases}$

a. (–7, –2), (7, 6) **b.** (–7, –2), (–7, 6) **c.** (7, –2), (7, 6) **d.** (–7, 4), (7, –4)

20. $\begin{cases} x^2 - 16 = 4y^2 \\ 2y - 2 = x \end{cases}$

a. $\left(-5, -\frac{3}{2}\right)$ **b.** $\left(-5, \frac{3}{2}\right)$ **c.** $\left(5, -\frac{3}{2}\right)$ **d.** $\left(5, \frac{3}{2}\right)$

21. Which is not a solution of the system $\begin{cases} 4x^2 - y = 3 \\ 8x^2 - y^2 = -9 \end{cases}$?

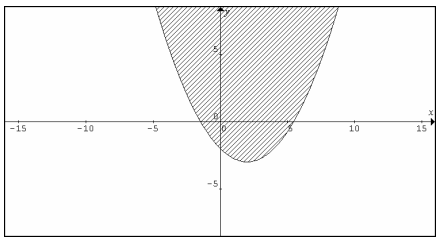
a. $\left(-\sqrt{2}, 5\right)$ **b.** $\left(\sqrt{2}, 5\right)$ **c.** $(0, -3)$ **d.** (0, 3)

22. What are the maximum number of real solutions for a system of equations for which the graphs are a circle and a hyperbola?

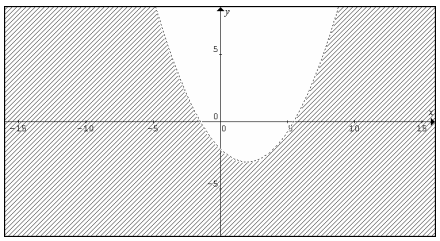
a. 0 **b.** 0 **c.** 4 **d.** 6

23. Match the graph to the inequality. $x^2 + 4x \geq 4y + 8$

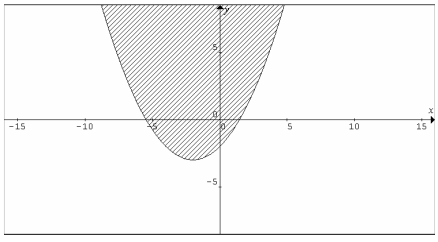
a.



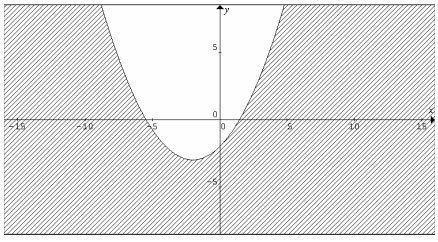
b.



c.



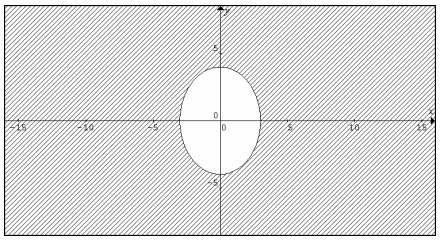
d.



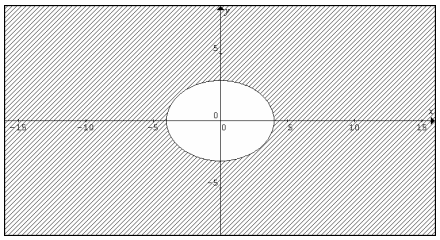
Chapter 10 Test Form D *cont'd*

24. Match the graph to the inequality. $16x^2 + 9y^2 \geq 144$

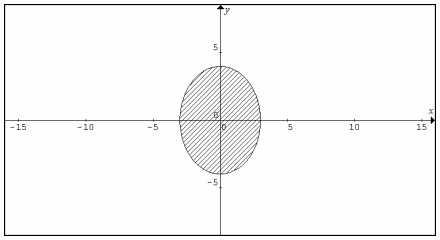
a.



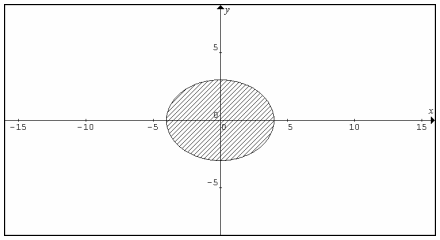
b.



c.

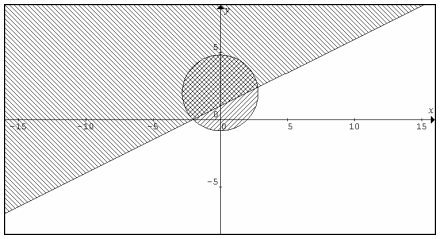


d.

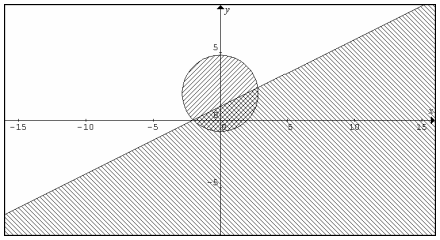


25. Match the system to the graph. $\begin{cases} x^2 + y^2 + 2x \leq 4 + 4y \\ x - 2y + 2 \geq 0 \end{cases}$

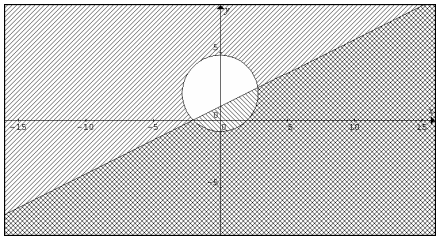
a.



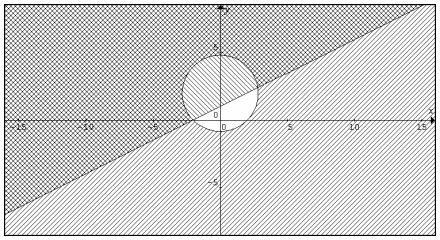
b.



c.



d.



Name: **Date:**
Instructor: **Section:**

Chapter 10 Test Form E

Circle the correct answer.

1. Give the vertex and direction of the parabola $x^2 + y - 12x + 34 = 0$.

a. (6, 2); opens to the left
b. (−6, −2); opens downward
c. (6, 2); opens downward
d. (−6, −2); opens to the left

2. Find the equation of the circle with center (−2, 1) and radius 5.

a. $x^2 + y^2 - 4x + 2y - 20 = 0$
b. $x^2 + y^2 + 4x - 2y - 20 = 0$
c. $x^2 + y^2 - 4x + 2y = 0$
d. $x^2 + y^2 + 4x - 2y = 0$

Use the following equation for problems 3 and 4. $x = -2(y-3)^2 + 5$

3. Find the vertex of the parabola.

a. (5, 3) **b.** (−5, −3) **c.** (3, 5) **d.** (−3, −5)

4. The graph of the parabola would open

a. upward **b.** downward **c.** right **d.** left

Identify the graph of each function.

5. $y^2 - 4y + x = -11$

a. parabola opening upward
b. parabola opening to the left
c. parabola opening downward
d. parabola opening to the right

6. $x^2 - 10x + 3 = y$

a. parabola opening upward
b. parabola opening downward
c. parabola opening to the right
d. parabola opening to the left

7. $16y^2 = 16x^2 + 10x - 3$

a. parabola **b.** circle **c.** ellipse **d.** hyperbola

8. $3y^2 + x + 2y - 6 = 0$

a. parabola **b.** circle **c.** ellipse **d.** hyperbola

Chapter 10 Test Form E *cont'd*

9. $6x^2 - 2x = 4y - 6y^2 + 9$

a. parabola **b.** circle **c.** ellipse **d.** hyperbola

10. $6(x+1)^2 + 4(y-2)^2 = 36$

a. parabola **b.** circle **c.** ellipse **d.** hyperbola

11. $6x^2 + 2x - y^2 + 3y = 14$

a. parabola **b.** circle **c.** ellipse **d.** hyperbola

12. $y + 3x^2 - 4x - 6 = 0$

a. parabola **b.** circle **c.** ellipse **d.** hyperbola

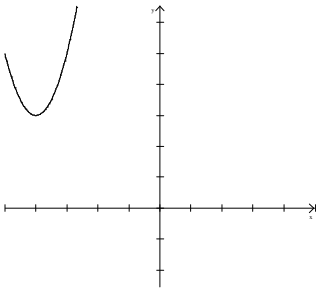
13. $3x^2 + 6x = 2 - y^2$

a. parabola **b.** circle **c.** ellipse **d.** hyperbola

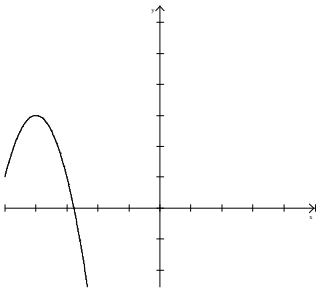
Match the graph to the equation.

14. $x = -2(y-3)^2 - 4$

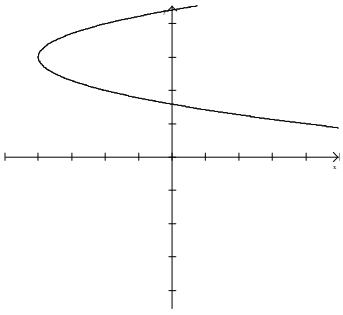
a.



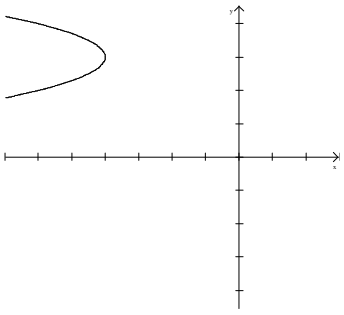
b.



c.



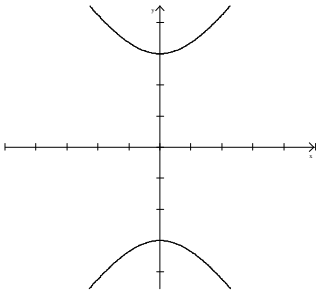
d.



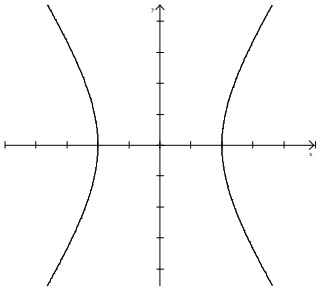
Chapter 10 Test Form E *cont'd*

15. $9y^2 - 4x^2 = 36$

a.



b.



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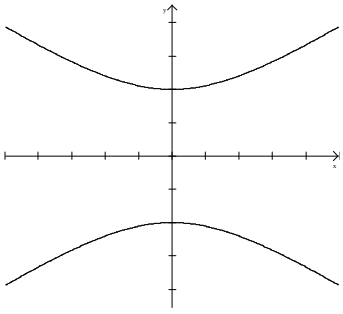
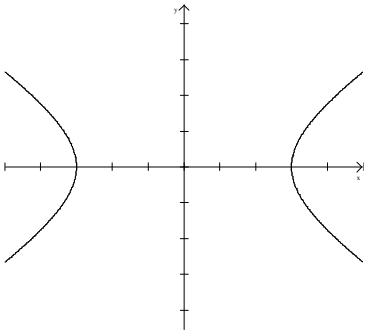
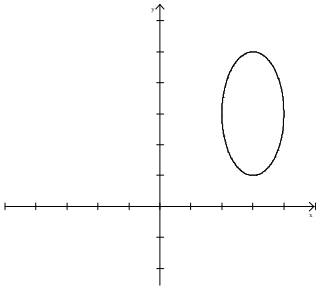


d.



16. $4x^2 + y^2 - 24x - 6y + 41 = 0$

a.



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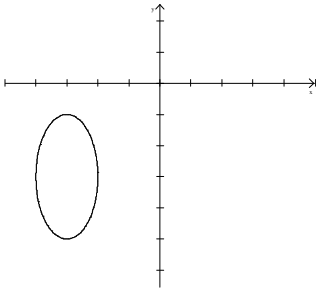

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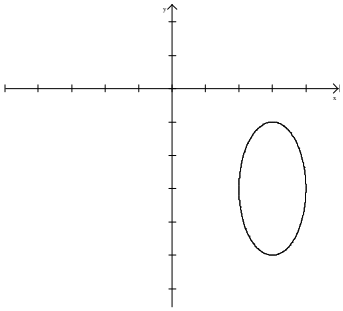
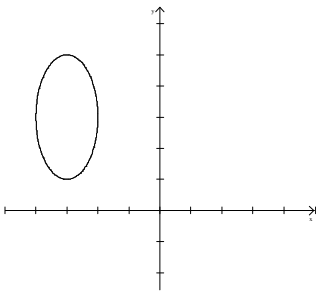


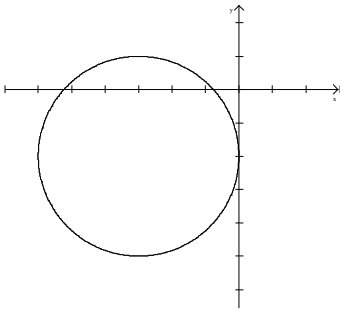
d.



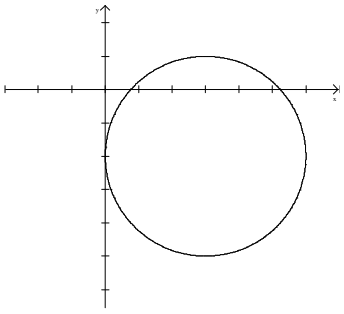
Chapter 10 Test Form E *cont'd*

17. $x^2 + y^2 - 6x - 4y + 4 = 0$

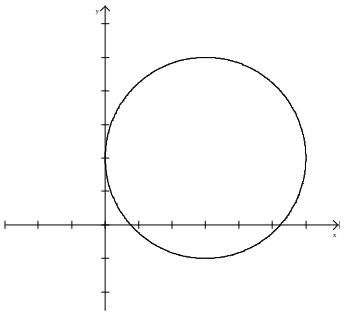
a.



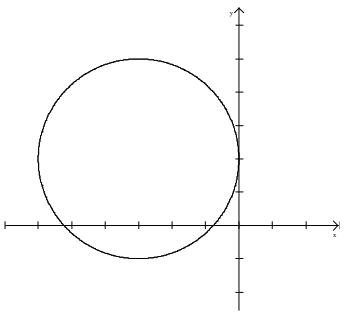
b.



c.



d.



Solve each system on equations.

18. $\begin{cases} (x-2)^2 + (y+3)^2 = 146 \\ 11x - 5y = 37 \end{cases}$

a. (−3, −14), (−3, 8) **b.** (−3, −14), (7, 8) **c.** (−3, 8), (7, 8) **d.** (7, 8), (12, 19)

19. $\begin{cases} x + 3(y+2)^2 = 4 \\ x + 3y = -8 \end{cases}$

a. (−8, 0), (1, −3) **b.** (−8, 0), (−8, −4) **c.** (1, −3), (7, −5) **d.** (7, −5), (−8, −4)

20. $\begin{cases} x^2 + y^2 = 4 \\ y = x^2 - 9 \end{cases}$

a. $\left(1, \sqrt{3}\right)$ **b.** (3, 0) **c.** Ø **d.** (−3, 0)

Chapter 10 Test Form E *cont'd*

21. Which is a solution of the system $\begin{cases} 2x^2 - 6y^2 + 3 = 0 \\ 4x^2 + 3y^2 - 4 = 0 \end{cases}$?

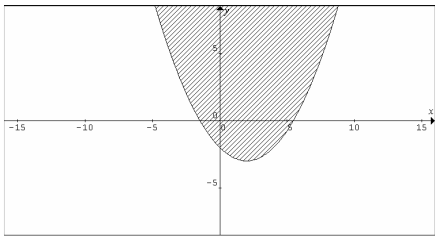
a. $\left(-\sqrt{2}, \sqrt{6}\right)$ **b.** $\left(\frac{\sqrt{3}}{2}, \frac{\sqrt{6}}{2}\right)$ **c.** $\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{6}}{3}\right)$ **d.** $\left(-\frac{\sqrt{2}}{6}, \frac{\sqrt{6}}{3}\right)$

22. What are the maximum number of real solutions for a system of equations for which the graphs are a circle and a parabola?

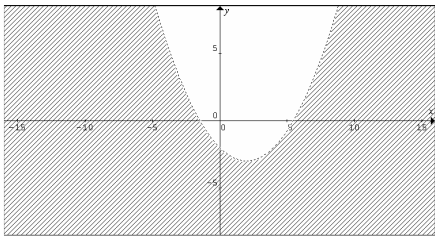
a. 2 **b.** 4 **c.** 1 **d.** 3

23. Match the graph to the inequality. $4y - x^2 \geq 4x - 8$

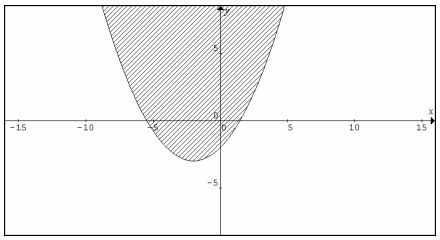
a.



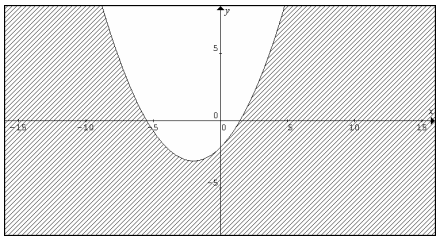
b.



c.



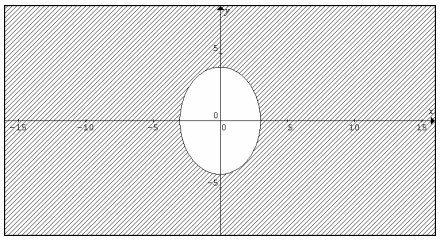
d.



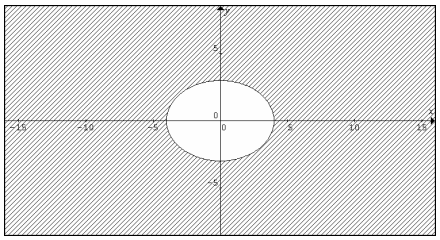
Chapter 10 Test Form E *cont'd*

24. Match the graph to the inequality. $16x^2 + 9y^2 \le 144$

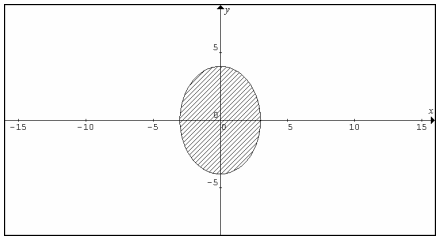
a.



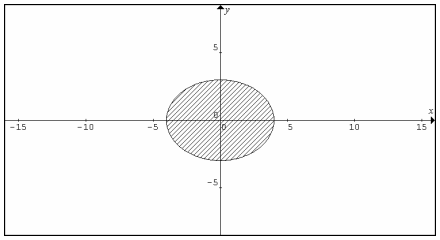
b.



c.

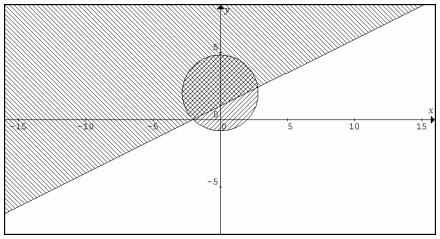


d.

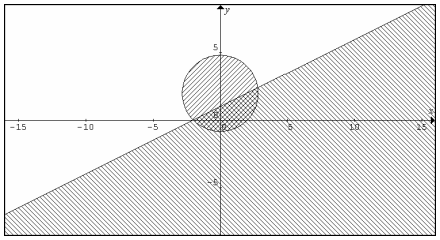


25. Match the system to the graph. $\begin{cases} 3x^2 + 16y \ge 48 \\ 4x + 7y \le 5 \end{cases}$

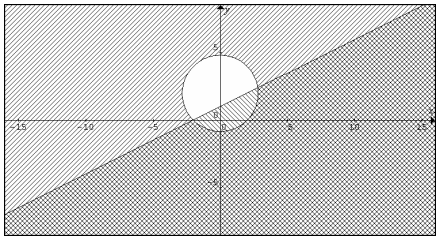
a.



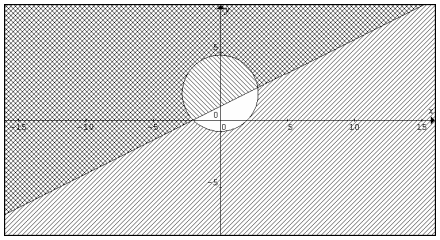
b.



c.



d.



Name: **Date:**
Instructor: **Section:**

Chapter 10 Test Form F

Circle the correct answer.

1. Give the equation of the circle with center (−2, 2) and radius 2.

a. $x^2 + y^2 + 4x - 4y + 4 = 0$ **b.** $x^2 + y^2 - 4x + 4y + 4 = 0$
c. $x^2 + y^2 + 4x - 4y + 6 = 0$ **d.** $x^2 + y^2 - 4x + 4y + 6 = 0$

2. Find the center and radius of the circle. $x^2 + y^2 + 4x + 2y - 4 = 0$

a. (−2, −1); 9 **b.** (−2, −1);3 **c.** (2, 1); 9 **d.** (2, 1); 3

Use the following equation for problems 3 and 4. $y = -2(x+5)^2 - 6$

3. Find the vertex of the parabola.

a. (5, −6) **b.** (−5, 6) **c.** (−5, −6) **d.** (5, 6)

4. The graph of the parabola would open

a. upward **b.** downward **c.** right **d.** left

Identify the graph of each function.

5. $6y^2 + 24y - x + 12 = 0$

a. parabola opening upward **b.** parabola opening downward
c. parabola opening to the right **d.** parabola opening to the left

6. $3x^2 + y - 12x = 0$

a. parabola opening upward **b.** parabola opening downward
c. parabola opening to the right **d.** parabola opening to the left

7. $6x^2 + 3x = 4y - 5y^2 + 11$

a. parabola **b.** circle **c.** ellipse **d.** hyperbola

8. $3x^2 = 7 - 2y - 3y^2$

a. parabola **b.** circle **c.** ellipse **d.** hyperbola

Chapter 10 Test Form F *cont'd*

9. $3x^2 - 6y^2 + 4x - 5y + 16 = 0$

a. parabola **b.** circle **c.** ellipse **d.** hyperbola

10. $4x^2 + 6x - 2y = 7$

a. parabola **b.** circle **c.** ellipse **d.** hyperbola

11. $y^2 - 2y + 6 = x$

a. parabola **b.** circle **c.** ellipse **d.** hyperbola

12. $x^2 + 5x = 3 - 2y^2$

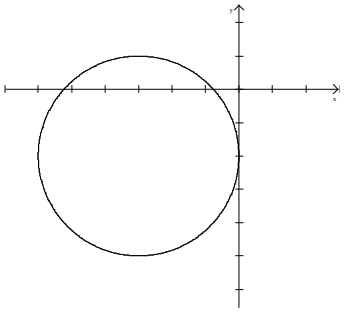
a. parabola **b.** circle **c.** ellipse **d.** hyperbola

13. $2x^2 - 2y^2 + 3x - 4y + 16 = 0$

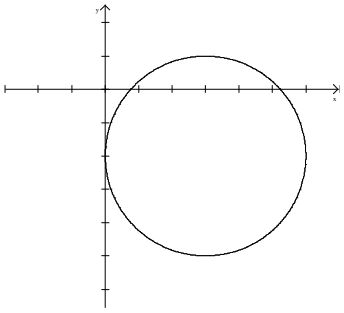
a. parabola **b.** circle **c.** ellipse **d.** hyperbola

14. $x^2 + y^2 + 6x - 4y + 4 = 0$

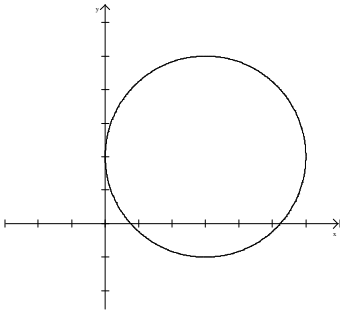
a.



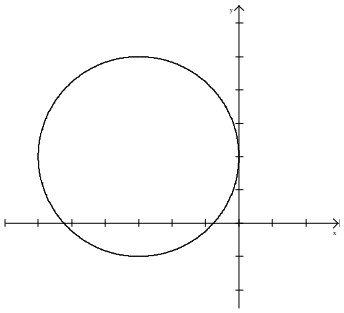
b.



c.



d.

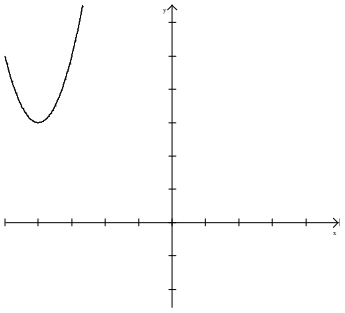


Chapter 10 Test Form F *cont'd*

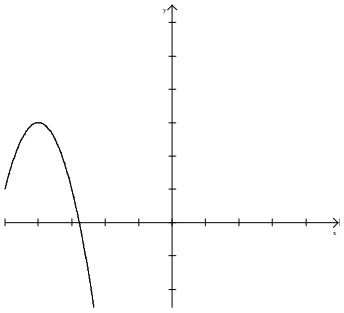
Match the graph to the equation.

15. $y = -2(x+4)^2 + 3$

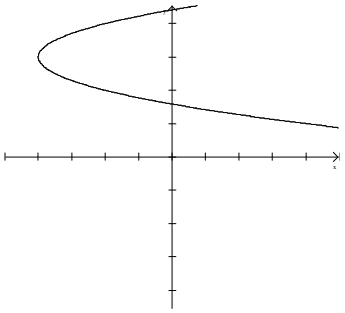
a.



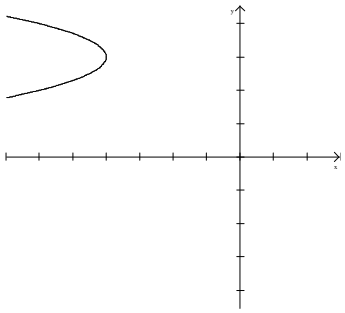
b.



c.

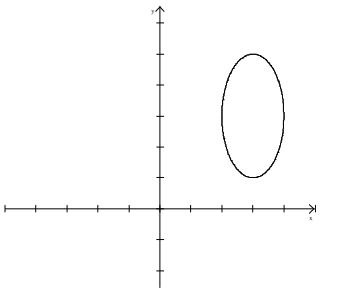


d.

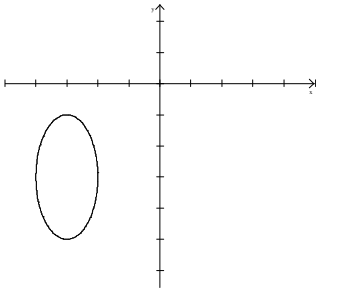


16. $4x^2 + y^2 - 24x + 6y + 41 = 0$

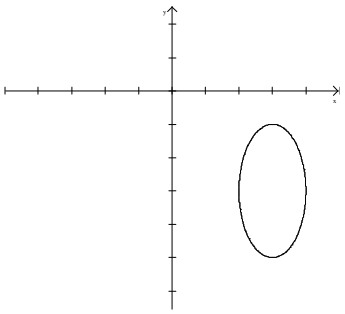
a.



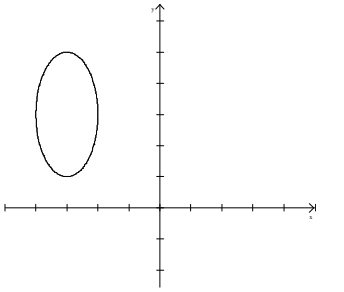
b.



c.



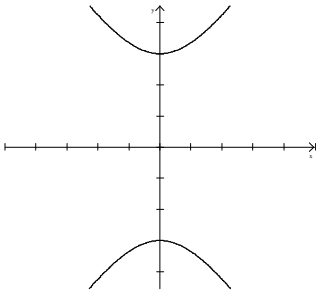
d.



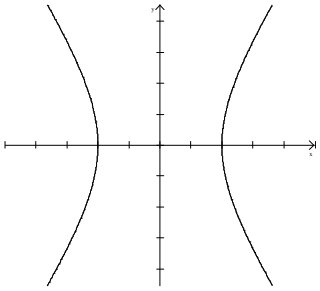
Chapter 10 Test Form F *cont'd*

17. $4x^2 - 9y^2 = 36$

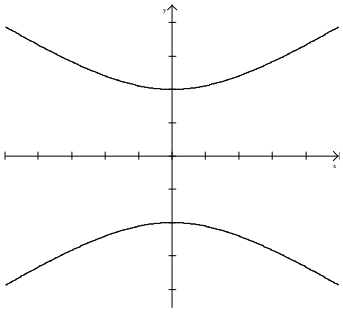
a.



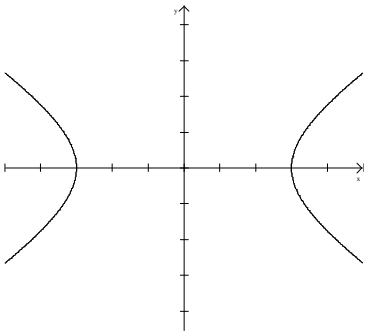
b.



c.



d.



Solve each system on equations.

18. $\begin{cases} x^2 + y^2 = 25 \\ x^2 + 3(y-8)^2 = 57 \end{cases}$

a. (−3, 4), (3, −4) **b.** Ø **c.** (−3, 12), (3, 4) **d.** (−3, 4), (3, 4)

19. $\begin{cases} 3x^2 + 6x - y = -6 \\ 3(x+1)^2 + (y-3)^2 = 12 \end{cases}$

a. Ø **b.** (−2, 6), (0, 6) **c.** (−2, 6), (−2, 0) **d.** (−2, 6), (1, 15)

Chapter 10 Test Form F *cont'd*

20. $\begin{cases} x^2 + y^2 = 13 \\ x^2 - y^2 = 5 \end{cases}$

a. (2, 3), (2, −3), (−2, 3), (−2, −3)
b. $\left(\sqrt{6},1\right),\left(-\sqrt{6},1\right),\left(\sqrt{6},-1\right),\left(-\sqrt{6},-1\right)$
c. (3, 2), (3, −2), (−3, −2), (−3, 2)
d. $\left(\sqrt{12},1\right),\left(-\sqrt{12},1\right),\left(\sqrt{12},-1\right),\left(-\sqrt{12},-1\right)$

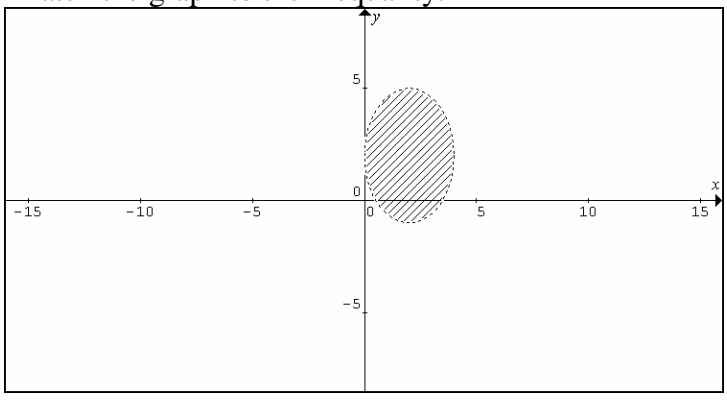
21. $\begin{cases} x^2 - 4y^2 = -7 \\ 3x^2 + y^2 = 31 \end{cases}$

a. (3, 2), (3, −2), (−3, 2), (−3, −2)
b. $\left(1,\sqrt{2}\right),\left(1,-\sqrt{2}\right),\left(-1,\sqrt{2}\right),\left(-1,-\sqrt{2}\right)$
c. $\left(\sqrt{5},4\right),\left(\sqrt{5},-4\right),\left(-\sqrt{5},4\right),\left(-\sqrt{5},-4\right)$
d. $\left(\sqrt{5},\sqrt{3}\right),\left(\sqrt{5},-\sqrt{3}\right),\left(-\sqrt{5},\sqrt{3}\right),\left(-\sqrt{5},-\sqrt{3}\right)$

22. What are the maximum number of real solutions for a system of equations for which the graphs are a ellipse and a parabola?

a. 0 **b.** 0 **c.** 4 **d.** 6

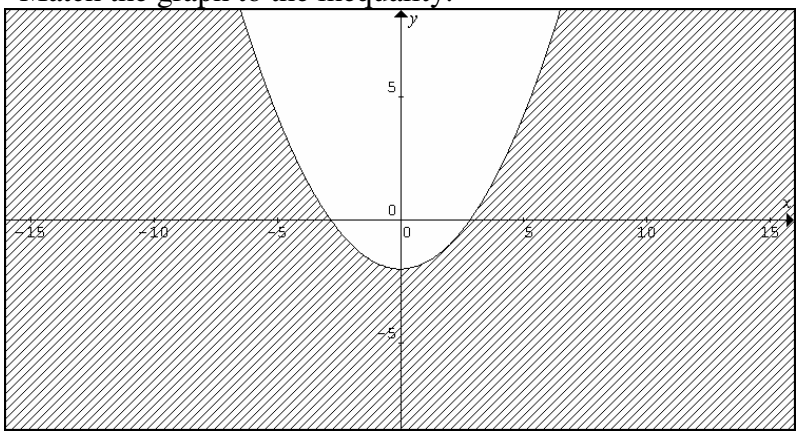
23. Match the graph to the inequality.



a. $4(x-2)^2+9(y-1)^2<36$ **b.** $9(x-2)^2+4(y-1)^2<36$
c. $4(x-2)^2+9(y-1)^2>36$ **d.** $9(x-2)^2+4(y-1)^2>36$

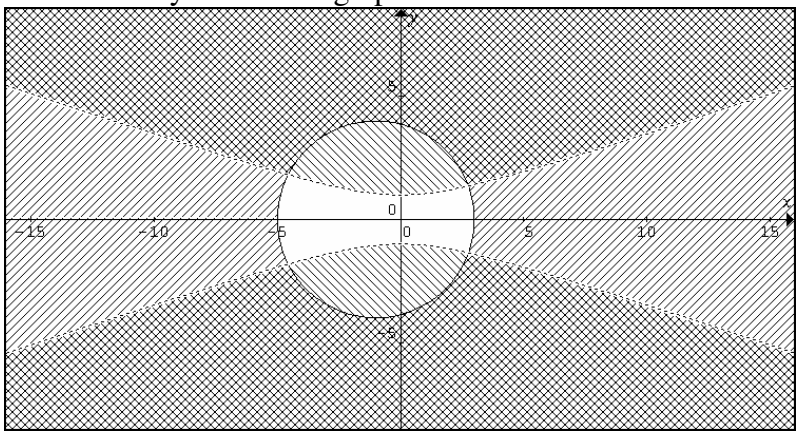
Chapter 10 Test Form F *cont'd*

24. Match the graph to the inequality.



a. $x^2 \le 4y + 8$ **b.** $x^2 \ge 4y + 8$ **c.** $x \le 4y^2 + 8$ **d.** $x \ge 4y^2 + 8$

25. Match the system to the graph.



a. $\begin{cases} x^2 + y^2 + 2x < 15 \\ 9y^2 \ge 9 + x^2 \end{cases}$ **b.** $\begin{cases} x^2 + y^2 + 2x \ge 15 \\ 9y^2 < 9 + x^2 \end{cases}$

c. $\begin{cases} x^2 + y^2 + 2x < 15 \\ 9y^2 > 9 + x^2 \end{cases}$ **d.** $\begin{cases} x^2 + y^2 + 2x \le 15 \\ 9y^2 > 9 + x^2 \end{cases}$