

① What are the missing math symbols here?

$$a! = a(a-1)(a-2)\dots 1$$

$$5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$$

$$3! = 3 \cdot 2 \cdot 1$$

$$(1 + 1 + 1)! = 6 \quad \checkmark$$

$$2 + 2 + 2 = 6 \quad \checkmark$$

$$(3 \cdot 3) - 3 = 6$$

$$4 + 4 - \sqrt{4} = 6$$

$$5 + (5 \div 5) = 6$$

$$6 \times 6 \div 6 = 6$$

$$7 - (7 \div 7) = 6$$

$$\sqrt[3]{8} + \sqrt[3]{8} + \sqrt[3]{8} = 6$$

$$(\sqrt{9} \cdot \sqrt{9}) - \sqrt{9} = 6$$

② What is the Horizontal and Vertical Asymptote of:

$$f(x) = \frac{3x^2 - 5}{x^2 - 9}$$

HA @ $y = \frac{3}{1}$ $\boxed{y = 3}$

VA @ $x^2 - 9 = 0$

$$(x+3)(x-3) = 0$$

$$x+3=0$$

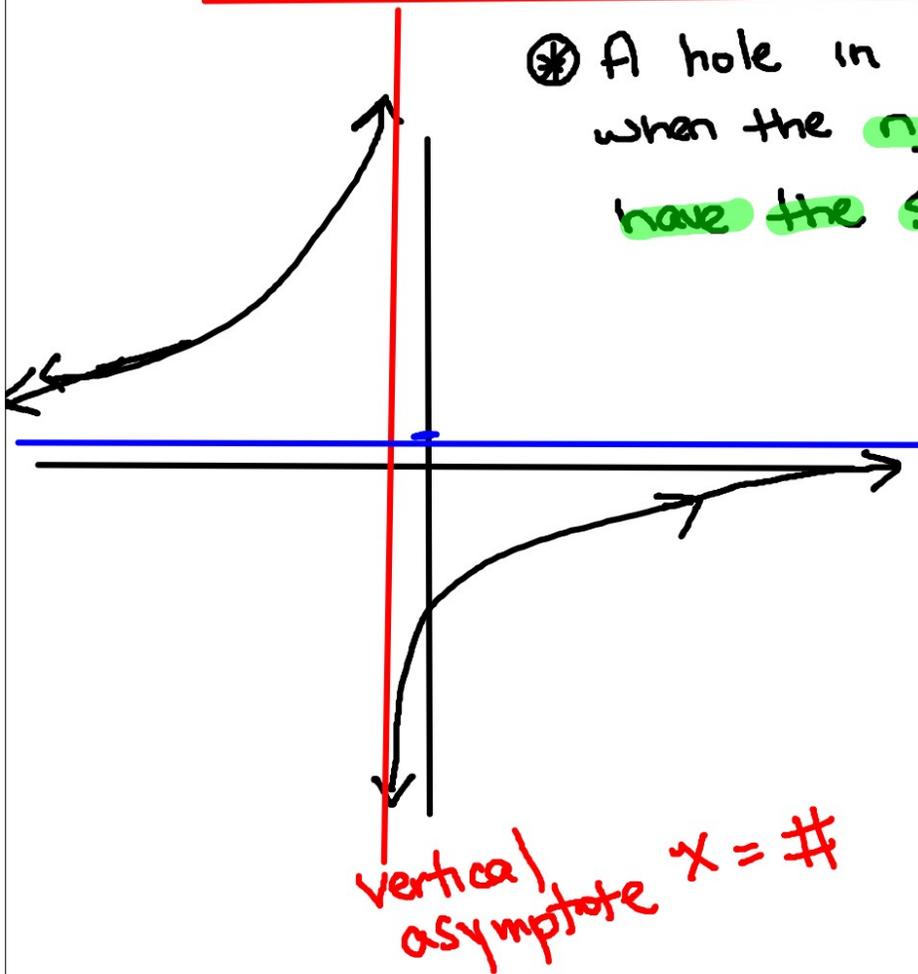
$$\boxed{x = -3}$$

$$x-3=0$$

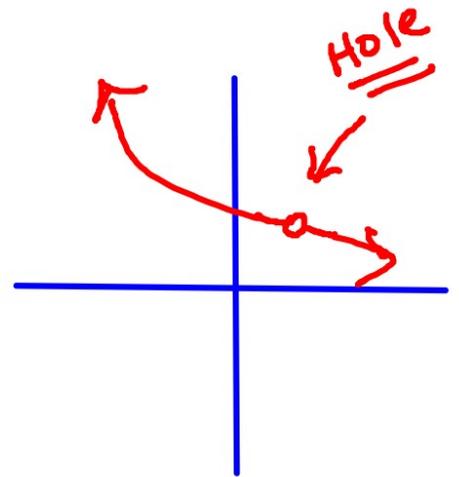
$$\boxed{x = 3}$$

Holes in Rational Functions

⊛ A hole in a function occurs when the numerator & denominator have the same factor (s).



Horizontal Asymptote $y = \#$



Ex: $f(x) = \frac{(x+3)}{(x+4)(x+3)}$ =

* They share $(x+3)$ so we set the common factor equal to zero and solve.

$$(x+3) = 0$$

$$x + \cancel{3} = 0$$

$\quad -\cancel{3} \quad -3$

$$x = -3$$

Has a hole
@ $x = -3$

$$f(x) = \frac{x^1 - 2}{x^2 - 7x + 10} = \frac{(x-2)}{(x-2)(x-5)}$$

$\Delta: 1$ (pointing to x^1)
 $\Delta: 2$ (pointing to x^2)
 ADD (pointing to $-7x$)
 MULT (pointing to $+10$)

Hok: $x - 2 = 0$
 $+2 \quad +2$

Hok \rightarrow $x = 2$

@

VA@

$x - 5 = 0$

$x = 5$

HA@

||

$y = 0$

$$f(x) = \frac{x^2 + 3x - 18}{x^2 + 7x + 6} = \frac{(x+6)(x-3)}{(x+6)(x+1)}$$

Annotations:

- For the numerator $x^2 + 3x - 18$:
 - x^2 is marked with a pink '2' and a red arrow labeled 'ADD' points to the '+' sign.
 - -18 is marked with a red arrow labeled 'MULT' points to the '-' sign.
- For the denominator $x^2 + 7x + 6$:
 - x^2 is marked with a pink '2'.
 - $+7x$ is marked with a red arrow labeled 'ADD' points to the '+' sign.
 - $+6$ is marked with a red arrow labeled 'MULT' points to the '+' sign.

VA @ $x+1 = 0$

$-1 \quad -1$

$x = -1$

Hole @ $x = -6$

$$x+6 = 0$$

$$-6 \quad -6$$

$x = -6$

HA @ $y = \frac{1}{1}$

$y = 1$

Find all holes & asymptotes

① $f(x) \Rightarrow \frac{3x^2 - 12}{x^2 + 4x + 4}$

GCF is x^2

$\frac{3(x^2 - 4)}{(x+2)(x+2)}$

Annotations: "ADD" with arrows pointing to $x^2 - 4$; "MULT" with an arrow pointing to the denominator.

$= \frac{3(x+2)(x-2)}{(x+2)(x+2)}$

HA @ $y = \frac{3}{1}$

HA @ $y = 3$

Hole @ $x+2=0$
 $-x-2$
 $x = -2$

V.A @ $x+2=0$
 $-2 \quad -2$
 $x = -2$

$f(x) = \frac{2x^2 + 3x + 1}{2x + 1}$

$= \frac{(2x+1)(x+1)}{2x+1}$

Annotations: Green circles around coefficients 2, 3, 1 in the numerator and 2, 1 in the denominator. A yellow highlight under $(2x+1)$ in both numerator and denominator. A question mark next to the cancelled denominator.

Hole: $2x+1=0$
 $\frac{2x}{2} = \frac{-1}{2}$
 $x = -\frac{1}{2}$

HA @ $y = 2$
 HA @ $y = 2$
 V.A
 NO HA

Diff of Squares →

$$f(x) = \frac{x^2 - 9}{(x-3)} = \frac{(x+3)(\cancel{x-3})}{(\cancel{x-3})}$$

(*)

Hole:

$$\begin{array}{r} x-3 = 0 \\ +3 \quad +3 \\ \hline \end{array}$$

Hole @

$$x = \underline{\underline{3}}$$

$$= (x+3)$$
$$3+3 = 6$$

Coordinates of
the Hole :

$$(\underline{\underline{3}}, \underline{\underline{6}})$$

$$f(x) = \frac{x^2 + 4x - 5}{x^2 + x - 2} = \frac{(x+5)(\cancel{x-1})}{(x+2)(\cancel{x-1})}$$

↑ ADD ↑ MULT
↑ ADD ↑ MULT

$$= \frac{x+5}{x+2} = \frac{1+5}{1+2} = \frac{6}{3} = 2$$

Hole * @ x = 1

$$\begin{array}{r} x-1 = 0 \\ +1 \quad +1 \\ \hline x = 1 \end{array}$$

* Coordinates of Hole
(1, 2)