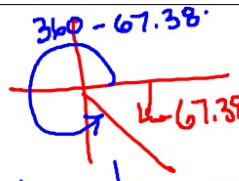
(r, θ)



DRILL

$\theta = [0, 360)$

① Switch to polar coordinates $(5, -12) \rightarrow (13, 292.62°)$

$r^2 = 5^2 + (-12)^2$ $\quad r = 13$ $\quad \tan\theta = \frac{-12}{5}$ $\quad \theta = -67.38°$

$r^2 = 169$

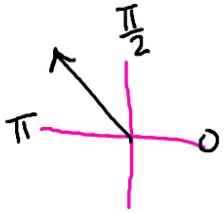
② Switch to rectangular coordinates $\left(-3, \frac{3\pi}{4}\right) \rightarrow \left(\frac{3\sqrt{2}}{2}, -\frac{3\sqrt{2}}{2}\right)$

Coordinate Conversion

$x = -3\cos\left(\frac{3\pi}{4}\right)$ $\quad y = -3\sin\left(\frac{3\pi}{4}\right)$

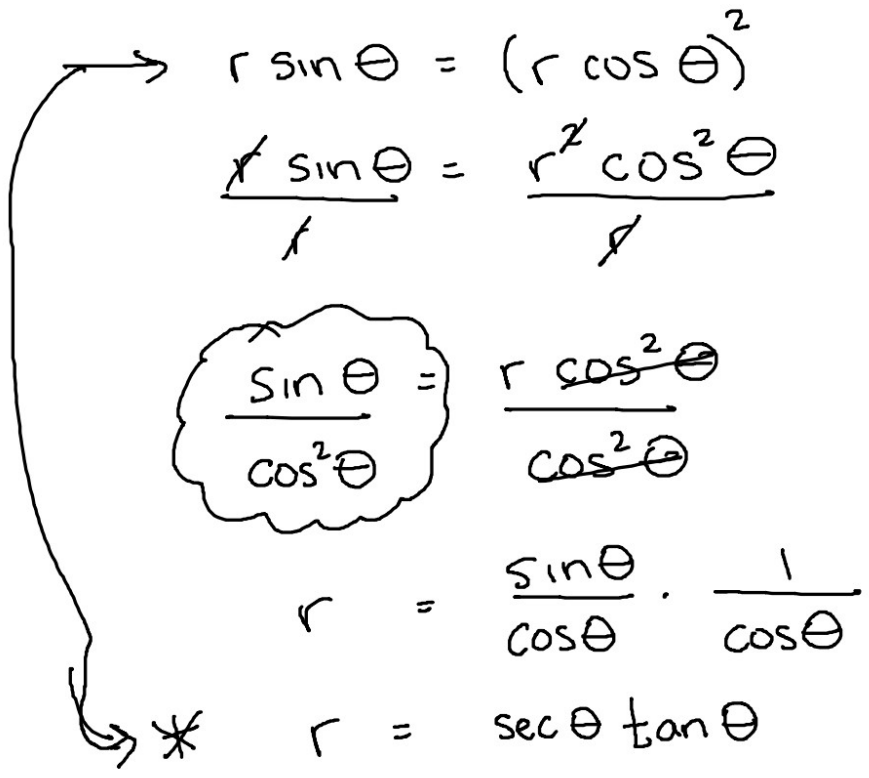
The polar coordinates (r, θ) are related to the rectangular coordinates (x, y) as follows.

Polar-to-Rectangular	*Rectangular-to-Polar*
$x = r\cos\theta$	$\tan\theta = \frac{y}{x}$
$y = r\sin\theta$	$r^2 = x^2 + y^2$



$x = -3\left(-\frac{\sqrt{2}}{2}\right)$

$y = -3\left(\frac{\sqrt{2}}{2}\right)$


$r \sin\theta = (r\cos\theta)^2$
$\frac{r \sin\theta}{r} = \frac{r^2 \cos^2\theta}{r}$
$\frac{\sin\theta}{\cos^2\theta} = \frac{r \cos^2\theta}{\cos^2\theta}$
$r = \frac{\sin\theta}{\cos\theta} \cdot \frac{1}{\cos\theta}$
$r = \sec\theta \tan\theta$

Example 5

$$\theta = \frac{\pi}{3}$$

$$\tan\theta = \tan\frac{\pi}{3}$$

$$x \cdot \frac{y}{x} = \sqrt{3} \cdot x \quad \rightarrow \quad \boxed{y = x\sqrt{3}}$$

$$\tan\theta = \frac{y}{x}$$

$$\tan\theta = \sqrt{3}$$

(c) $r = \sec\theta$ $\rightarrow \cos\theta\, r = \frac{1}{\cos\theta} \cdot \cos\theta$

$r\cos\theta = 1$ $\rightarrow$ $\boxed{x = 1}$

$x = r\cos\theta$ $\qquad$ $y = r\sin\theta$

Try It

① $x^2 + y^2 = 9$

$\{ \cos^2\theta + \sin^2\theta = 1$

$(r\cos\theta)^2 + (r\sin\theta)^2 = 9$

$r^2\cos^2\theta + r^2\sin^2\theta = 9$

$r^2(\cos^2\theta + \sin^2\theta) = 9$

$r^2(1) = 9$

$\sqrt{r^2} = \sqrt{9}$

$r = \pm 3$

Try It ② $\theta = \frac{\pi}{4}$

$\tan\theta = \frac{y}{x}$

$\tan\theta = \tan\frac{\pi}{4}$

$\frac{y}{x} = 1$

$x \cdot \frac{y}{x} = 1 \cdot x$

$y = x$

Try It ③ $r\cos\theta = 2$

$x = r\cos\theta$

$\downarrow$

$x = 2$