

Answers to Practice Test # 3 Math 165

$$\textcircled{1} \tan \theta \sin \theta + \cos \theta$$

$$= \frac{\sin \theta}{\cos \theta} \cdot \sin \theta + \cos \theta$$

$$= \frac{\sin^2 \theta}{\cos \theta} + \frac{\cos \theta (\cos \theta)}{\cos \theta}$$

$$= \frac{\sin^2 \theta + \cos^2 \theta}{\cos \theta}$$

$$= \frac{1}{\cos \theta}$$

$$= \boxed{\sec \theta}$$

$$\textcircled{2} \frac{\cos \theta}{1 + \sin \theta} + \frac{\sin \theta}{\cos \theta}$$

$$= \frac{(\cos \theta) \cos \theta}{(\cos \theta) (1 + \sin \theta)} + \frac{\sin \theta (1 + \sin \theta)}{\cos \theta (1 + \sin \theta)}$$

$$= \frac{\cos^2 \theta + \sin \theta + \sin^2 \theta}{\cos \theta (1 + \sin \theta)}$$

$$= \frac{1 (1 + \sin \theta)}{\cos \theta (1 + \sin \theta)}$$

$$= \frac{1}{\cos \theta} = \boxed{\sec \theta}$$

$$\textcircled{3} \frac{\csc^2 \theta - \cot^2 \theta}{\sin^2 \theta} \cdot \csc^2 \theta$$

$$\frac{\left(\frac{1}{\sin^2 \theta} - \frac{\cos^2 \theta}{\sin^2 \theta} \right)}{\sin^2 \theta} = \csc^2 \theta$$

$$\frac{\left(\frac{1 - \cos^2 \theta}{\sin^2 \theta} \right)}{\sin^2 \theta} = \frac{(\sin^2 \theta)}{(\sin^2 \theta) \sin^2 \theta} = \frac{1}{\sin^2 \theta} = \boxed{\csc^2 \theta}$$

$$\textcircled{4} \quad \sec x - \tan x \sin x = \cos x$$

$$\rightarrow \frac{1}{\cos x} - \frac{\sin x}{\cos x} \cdot \sin x = \cos x$$

$$\rightarrow \frac{1 - \sin^2 x}{\cos x} = \cos x$$

$$\rightarrow \frac{\cos^2 x}{\cos x} = \cos x$$

$$\cos x = \cos x$$

$$\textcircled{5} \quad \begin{aligned} \cos^2 \theta - \sin^2 \theta &= 1 - 2\sin^2 \theta \\ (1 - \sin^2 \theta) - \sin^2 \theta &= 1 - 2\sin^2 \theta \\ 1 - 2\sin^2 \theta &= 1 - 2\sin^2 \theta \end{aligned}$$

$$\begin{aligned} \textcircled{6} \text{ a) } \sin 105^\circ &= \sin(60^\circ + 45^\circ) \\ &= \sin 60^\circ \cos 45^\circ + \cos 60^\circ \sin 45^\circ \\ &= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) \\ &= \frac{\sqrt{6} + \sqrt{2}}{4} \end{aligned}$$

$$\begin{aligned} \text{b) } \cos\left(\frac{\pi}{12}\right) &= \cos\left(\frac{\pi}{3} - \frac{\pi}{4}\right) \\ &= \cos \frac{\pi}{3} \cos \frac{\pi}{4} + \sin \frac{\pi}{3} \sin \frac{\pi}{4} \\ &= \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2} + \sqrt{6}}{4} \end{aligned}$$

$$\textcircled{7} \quad \sin x + \sqrt{2} = -\sin x$$

$$\frac{2 \sin x}{2} = -\frac{\sqrt{2}}{2}$$

$$\sin x = -\frac{\sqrt{2}}{2}$$

$$x = \frac{5\pi}{4}, \frac{7\pi}{4}$$

$$\textcircled{8} \quad 2\sin^2 x + 2\sin x + 3 = 3\sin x + 4$$

$$2\sin^2 x - \sin x - 1 = 0$$

$$(2\sin x + 1)(\sin x - 1) = 0$$

$$\sin x = -\frac{1}{2} \quad \sin x = 1$$

$$x = \frac{7\pi}{6}, \frac{11\pi}{6}, \frac{\pi}{2}$$

$$\textcircled{9} \quad 2\cos(3t) - 1 = 0$$

$$\cos(3t) = \frac{1}{2}$$

$$\cos \theta = \frac{1}{2}$$

$$3t = \frac{\pi}{3} \quad \text{or} \quad 3t = \frac{5\pi}{3}$$

$$t = \frac{\pi}{9} \quad \text{or} \quad t = \frac{5\pi}{9}$$

$$\theta = \frac{\pi}{3}, \frac{5\pi}{3}$$

$$\textcircled{10} \quad \sec^2 x - 2\tan x = 4$$

$$(\tan^2 x + 1) - 2\tan x - 4 = 0$$

$$\tan^2 x - 2\tan x - 3 = 0$$

$$(\tan x + 1)(\tan x - 3) = 0$$

$$\tan x = -1 \quad x = \frac{3\pi}{4}, \frac{7\pi}{4}$$

$$\tan x = 3 \quad \text{Calculator} \\ x = \tan^{-1}(3)$$

Not on test