Chi-Squared Goodness of Fit Test (G.O.F)

* Used when we want to compare multiple %'s (proportions) to a given model. [Categorical Data]

1) Population

* 2) Model: Each category with the given percent/proportion (Given in the problem)

3) Type of Test: Chi-Squared G.O.F

4) Conditions:
   - Random Sample ✓
   - 10n < Pop. Size ✓
   - No expected counts less than 5. ✓

⑤ Hypothesis: Ho: The model is a good fit (context of problem)

Ha: The model is not a good fit

⑥ Calculations / Formula / Statistics & p-value ($\chi^2 \Rightarrow$ chi-squared)

$$\chi^2 = \underset{\text{(sum)}}{\sum} \frac{[(\text{obs}) - (\text{exp})]^2}{\text{exp}}$$

obs: observed value (what we saw)

exp: expected value

p-value

Expected Values: (model %'s)(sample size (n))

Expected Values have a sum of "n"

Degrees of Freedom: (d.f) = # of categories − 1

(7) Conclusion: (based on the p-value)

* If the p-value is less than 5%
  we reject the Ho.

* If the p-value is greater than 5%
  we fail to reject the Ho.

A professor tells the class on the first day that the distribution for grades in all of his classes over the past 20 years, has been as shown:

21% A's, 32% B's, 25% C's, 14% D's, and 8% F's *

At the end of the semester the class of 40 students feels the professor was lying to them when he originally told them his normal break down of grades due to the fact that out of the 40 students there were 7 A's, 10 B's, 14 C's, 4 D's and 5 F's. Does this data support the class's claim that the professor was probably lying to them at the beginning of the semester?

① Population: All students taking the professor's class

② Model: 21% A's, 32% B's, 25% C's, 14% D's, 8% F's

③ Type of Test: Chi-Squared G.O.F

④ Conditions: Random Sample ✓ No expected counts less than 5. ✓ *

10n < Pop. Size ✓

⑤ Ho: The professor's grade distribution matches the stated model.

Ha: " " does not " "

1) A professor tells the class on the first day that the distribution for grades in all of his classes over the past 20 years, has been as shown:

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⑥ Formula: → $\chi^2 = \sum^{\text{sum}} \frac{(OBS - EXP)^2}{EXP}$ → EXP values: (%)(n) n = 40

D.F = 5 − 1 = 4 (41.76%)

$\chi^2 \approx 3.92$ p-value ≈ .4176

CATEGORY	OBS	→ EXP	$\frac{(O-E)^2}{E}$
A	7	8.4	→ ...
B	10	12.8	→ ...
C	14	10	
D	4	5.6	
F	5	3.2	

* Since our p-value is greater than 5% we fail to reject the Ho, which means we do not have enough evidence to suggest the professor was lying about the grade distribution.

Pop	30% ✓
Rap	20%
Country	20%
Classical	30% ✓

$n = 80$ (songs)

Pop = 30
rap = 25
country = 15
classical = 10

OBS

Pop = 24   80(.3)
rap = 16   80(.2)
country = 16   80(.2)
classical = 24   80(.3)

EXP

EXP = (n)(%'s)

30% = .30

χ^2 : 14.79

D.F = 3

p-value: .002

1) Sara hears an advertisement for an award show where they make a claim that 20% of the guests are famous athletes, 30% of the guests are famous singers, 10% of the guests are well known politicians, 25% of the guests are famous actors/actresses and the remaining guests are non-celebrities. Sara decides to go and check it out, she takes a random sample of $\boxed{100}$ people at the awards show and finds out that 14 are famous athletes, 36 are famous singers, 2 are well know politicians and 30 are famous actors/actresses and the others were non-celebrities. Conduct the appropriate tests to see if there is evidence that would suggest the claim made in the advertisement is false.

$n = 100$

CATEGORY	MODEL (100-85%)	OBS (100-82)	EXP $n(\%)$
· Athletes :	.20%	14	.20·
· Singers :	.30%	36	.30·
· Politicians :	.10%	2	.10·
· Act/Actress :	.25%	30	.25·
· Non-Celeb :	$\boxed{.15\%}$	18	.15·

$X^2 = 11$ D.F: 4

P-value = .0266

Pop: All people at an awards show

Chi-Squared G.O.F

Random Sample ✓

10n < Pop size ✓

No EXP. counts less than 5.

1) Sara hears an advertisement for an award show where they make a claim that 20% of the guests are famous athletes, 30% of the guests are famous singers, 10% of the guests are well known politicians, 25% of the guests are famous actors/actresses and the remaining guests are non-celebrities. Sara decides to go and check it out, she takes a random sample of 100 people at the awards show and finds out that 14 are famous athletes, 36 are famous singers, 2 are well know politicians and 30 are famous actors/actresses and the others were non-celebrities. Conduct the appropriate tests to see if there is evidence that would suggest the claim made in the advertisement is false.

Ho: The model in the advertisement is $\boxed{\text{accurate}}$ p-value (.0266)

Ha: " " is NOT accurate

$$\chi^2 = \sum \frac{(OBS - EXP)^2}{EXP}$$

Since our p-value is less than 5%. we reject the Ho, which means the evidence suggests that the advertised percents were not accurate.

1) 100 Randomly selected Americans over the age of 18 were asked how many hours a night they sleep, and the selected Americans had a mean sleep time of 6.8 hours with a standard deviation of 0.9 hours. An MIT professor believed that people in Italy over the age of 18 sleep more on average then people over the age of 18 in America. So he also asked 120 randomly selected Italian citizens over the age of 18 how many hours a night they sleep. The selected Italians had a mean of 7.1 hours of sleep and a standard deviation of 0.6 hours. Is there enough evidence to support the professor's claim?

2-Sample T Test

Pop_A : All Americans over the age of 18

Pop_I : All Italians over the age of 18

$H_0: \mu_A = \mu_I$

$H_a: \mu_A < \mu_I$

$$t = \frac{(\bar{x}_A - \bar{x}_I) - 0}{\sqrt{\frac{S_A^2}{n_A} + \frac{S_I^2}{n_I}}}$$

$\bar{x}_A$: 6.8
S_A: .9
n_A: 100

$\bar{x}_I$: 7.1
S_I: .6
n_I: 120

S.E ≈ .1054
DF ≈ 166.87

T-stat
-2.85
p-value ≈ .0025

Parameter: (μ_A & μ_I) the average # of hours of sleep.

Conditions: Random Samp A ✓
$10 n_A < Pop_A$ ✓
$n_A \geq 30$ ✓

Random Samp I ✓
$10 n_I < Pop\ I$ ✓
$n_I \geq 30$ ✓

1) 100 Randomly selected Americans over the age of 18 were asked how many hours a night they sleep, and the selected Americans had a mean sleep time of 6.8 hours with a standard deviation of 0.9 hours. An MIT professor believed that people in Italy over the age of 18 sleep more on average then people over the age of 18 in America. So he also asked 120 randomly selected Italian citizens over the age of 18 how many hours a night they sleep. The selected Italians had a mean of 7.1 hours of sleep and a standard deviation of 0.6 hours. Is there enough evidence to support the professor's claim?

Since our p-value is less than 5%. we reject the Ho, which means the evidence suggests the mean amount of sleep for people over the age of 18 in America is less than the average amount of sleep for people over the age of 18 in Italy.

1) A player wanted to see if their hitting coach was actually helping their batting average, so he did a statistical analysis to test the hypothesis that the coach was increasing his batting average. Before he hired the hitting coach the player got a hit about 27% of the time. After he hired the hitting coach he randomly selected 160 at bats in which he had 48 hits, is this enough statistical evidence that the coach has helped the player?

1-Proportion Test

Conditions: Random Samp ✓ 10n < Pop size ✓ $np_0 \geq 10$ & $n(1-p_0) \geq 10$

Population: All at bats by a player

Parameter: (p) proportion of at bats the player gets a hit

$n = 160$

* $H_0: p = .27$

$H_a: p > .27$

$\hat{p} = \frac{48}{160}$

$$Z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$$

Since our p-value is greater than 5% we fail to reject the Ho, which means we do not have enough evidence to say the coach is helping the batter.

Proportion	Count	Total	Sample Prop.	Std. Err.	Z-Stat	P-value
p	48	160	0.3	0.035098077	0.85474769	0.1963

1) You are the manager of the packaging process at a cereal manufacturing plant. You want to determine if the cereal filling process is working properly. The process requires no corrective action if the correct amount of cereal per box is at least 368 grams. To study this, you decide to take a random sample of 45 boxes, weigh each one, and then evaluate the difference between the sample statistic and the hypothesized population parameter by comparing the mean weight from the sample to the expected population mean of 368 grams specified by the company. The sample mean is 372.5 and the sample standard deviation is 15 grams. Is there evidence that the weight is *different* from 368 grams?

1 1 - Sample t 4 Conditions: Random Samp ✓ 10n < Pop size ✓ $n \geq 30$ ✓

2 Population: All cereal boxes produced by a company

3 Parameter: (μ) average amount of cereal per box (in grams)

5 Ho: $\mu = 368$
Ha: $\mu \neq 368$

6 $\bar{x} = 372.5$ $n = 45$
$s = 15$

$$t = \frac{\bar{x} - \mu_0}{\left(\frac{s}{\sqrt{n}}\right)}$$

Mean	Sample Mean	Std. Err.	DF	T-Stat	P-value
μ	372.5	2.236068	44	2.0124612	0.0503

5.03%

Since the p-value is greater than 5% we fail to reject the Ho, which means we do not have enough evidence to suggest the mean weight of cereal per box is different (not equal) from 368 grams

1) Explain what a Type I error and a Type II error would be in the context of the problem and state which one you think would be worse and why? All court cases in the United States start by assuming the person is Not Guilty (Ho).

Scenario: A man is on trial for stealing a piece of jewelry from a store, the punishment if found guilty is 1 year in prison. (Jail)

Type I: Reject Ho when it is true : Innocent person went to jail (1-year in jail)

Type II: Fail to reject when it is false : Guilty person set free

1) What type of test would we conduct if we were trying to see if the average salary in the United States was more than $41,000 per year given a sample mean of $41,750 and it is known that the population standard deviation is known to be $1,500? (Pick the best choice below)

1- Sample Z
(pop S.D)

1) We are conducting a two proportion test to see if Males or Females are more likely to get a speeding ticket. We take a random sample of 30 Males and 25 Females and find out that 12 of the Males had received a speeding ticket and 8 of the Females had received a speeding ticket. Given this information check the conditions for the Hypothesis Test and state which conditions (if any) are not met. If one or more conditions are not met what would we do to fix the problem?

$\hat{p}_M = \frac{12}{30}$ $\hat{p}_F = \frac{8}{25}$

✓ Random Samp M ✓ Rand. Samp F

✓ $10 n_M < \text{Pop } M$ ✓ $10 n_F < \text{Pop } F$

speed

$\cancel{30}\left(\frac{12}{\cancel{30}}\right)$ $n_M \hat{p}_M \geq 10$ ✓ $\cancel{25}\left(\frac{8}{\cancel{25}}\right)$ $n_F \hat{p}_F \geq 10$

$\cancel{30}\left(\frac{18}{\cancel{30}}\right)$ $n_M(1-\hat{p}_M) \geq 10$ ✓ $\cancel{25} \cdot \frac{17}{\cancel{25}}$ $n_F(1-\hat{p}_F) \geq 10$ ✓

* Select more females until we had 10 with tickets *