Multiplying Complex Numbers

Recall FOIL: $(a+b)(c+d) = ac + ad + bc + bd$

$i^2 = -1$

I. Model Problems

In these examples we will multiply complex numbers

Example 1: Simplify $\boldsymbol{2i(3+7i)}$

Distribute $2i$.

$$2i(3+7i)$$
$$2i(3) + 2i(7i)$$
$$6i + 14i^2$$

Substitute -1 for i^2. Simplify.

$$6i + 14(-1)$$
$$6i - 14$$

Rewrite in standard form.

$$-14 + 6i$$

Answer: $\boldsymbol{2i(3+7i) = -14 + 6i}$

Example 2: Simplify $\boldsymbol{(5+2i)(6-4i)}$

Multiply with the FOIL method.

$$(5+2i)(6-4i)$$
$$5(6) + 5(-4i) + 2i(6) + 2i(-4i)$$
$$30 - 20i + 12i - 8i^2$$
$$30 - 8i - 8i^2$$

Substitute -1 for i^2. Simplify.

$$30 - 8i - 8(-1)$$
$$30 - 8i + 8$$
$$38 - 8i$$

Answer: $\boldsymbol{(5+2i)(6-4i) = 38 - 8i}$

II. Practice Problems

Simplify.

1. $6i(4-12i)$
2. $-3i(9-6i)$
3. $-11i(3+9i)$
4. $2.4i(18+12i)$
5. $-0.2i(16+8i)$
6. $(2+4i)(3+3i)$
7. $(3+5i)(4+4i)$
8. $(2-4i)(3+5i)$
9. $(-3+2i)(-3-4i)$
10. $(5-10i)(3+5i)$
11. $(-3-7i)(-12-2i)$
12. $(15+3i)(4-15i)$
13. $3i(4+2i)(2+5i)$
14. $-2i(3-7i)(4+2i)$
15. $2i(4-5i)(4+5i)$
16. $10(3-2i)(4+3i)$
17. $6i(7-10i)(7+10i)$
18. $(3+2i)^2$
19. $-(2+2i)^2$
20. $i(4+3i)^2$

IV. Answer Key

1. $72 + 24i$
2. $-18 - 27i$
3. $99 - 33i$
4. $-28.8 + 43.2i$
5. $1.6 - 3.2i$
6. $-6 + 18i$
7. $-8 + 32i$
8. $26 - 6i$
9. $17 + 6i$
10. $65 - 5i$
11. $22 + 90i$
12. $105 - 213i$
13. $-72 - 6i$
14. $-44 - 52i$
15. $82i$
16. $180 + 10i$
17. $894i$
18. $5 + 12i$
19. $-8i$
20. $24 + 7i$

Recognizing a pattern

21. 13
22. 41
23. 58
24. Complex conjugates do not have an 'i' term because the 'i' terms are additive inverse and always sum to zero much like there is no 'x' term when you multiply (x - 3)(x + 3) or (x-5)(x+5)

Challenge Problems

1. $(a - bi)(a + bi) = a^2 - abi + abi - b^2 i^2$
   $= a^2 + b^2(-1) = a^2 + b^2$
2. $i^2 = -1; -8(-1) = 8$
2. $a = b$
3. $a = 1$
4. Forgot to multiply outside times outside and inside times inside