



Day 3: Paper Folding

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Introduction

In day 3 we recommend another favorite activity, which comes from Mark Driscoll. The activity encourages students and teachers to engage in visual, creative thinking. We have coupled Mark's activity with a strategy for teaching students to reason and be convincing, two important mathematical practices. We are also sharing a dot card number talk, that can be used at any point in the lesson.

Video

The video explains to students that depth is more important than speed in math, and shares the story of a famous mathematician. This video really encourages learners who may have got the wrong ideas about math.

Activity: Paper Folding

This is an activity that challenges any learner. It starts off with fairly easy questions that become more difficult as the activity progresses. When I have given it to teachers some have stayed up into the night trying to solve the last question! One thing I love about this activity is that even though the later questions are difficult, students know everything they need to know to solve them. This is a great opportunity to encourage students with positive messages, eg: "If it is difficult that means your brain is growing, you know everything you need to know to solve it, you just need to think hard"

I also really like this activity for teaching students to reason and justify, and be convincing, which are really important mathematical practices. I have combined the paper folding activity with a pedagogical strategy that I learned from Cathy Humphreys and that students enjoy, of learning to convince a skeptic, as I explain below.

Every student needs their own blank piece of square paper. It is easy to make a square out of a piece of 8.5 x 11 paper. If you have time in your lesson you can give the students the regular paper and ask them to make a square. This does take about 5 minutes for students to do this so you may want to prepare the squares ahead of time. Explain to students that in this activity they will be folding paper, making shapes and convincing someone that their shape is correct. A great opportunity to model what it means to be convincing comes about if you ask all students to find a solution to the first question, then invite one student to defend their solution with you. When they show you their solution, push them to be really convincing, explaining that this is what it is to be mathematical.

The first question asks students to construct a square with exactly $\frac{1}{4}$ the area of the original square and convince someone that it is a square and has $\frac{1}{4}$ of the area. This is a conversation I had with a student who shared their solution.

Student: This is a square, here

JB: How do you know it is a square?

Student: Because I folded it into 4 pieces.

JB: How does that convince you it is a square?

Student: The 4 pieces are equal.

JB: Why does that mean they are a square?



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Student: They are squares because they have 90 degree angles and their sides are all the same length. (student shows you their paper to show there are 4 small squares all the same)

JB: Ok, you have convinced me it is a square. How do you know that the square is $\frac{1}{4}$ the area of the original square?

Student: There are four squares that are the same size and together they are the same size as the big square

JB: How do you know they are the same size as the big square?

Student: Because the four squares fit into the big square exactly.

JB: Thanks, you have convinced me!

You can probably ask other questions to push your student to be convincing, what is really important is to get students to defend each solution fully, not making any statements that are not fully justified.

After this part of the activity tell students you want them to play the role of a skeptic with each other. Tell them there are three levels of being convincing. You can

- Convince yourself
- Convince a friend
- Convince a skeptic

It is easy to convince yourself and may be easy to convince a friend but convincing a skeptic is hard. Skeptics are very helpful as they push students to justify and reason. (see Cathy Humphreys use this strategy in Boaler & Humphreys, 2005).

For each of the remaining questions ask students to work in pairs and have one student answer the question with their paper and the other be a skeptic. After each question they switch roles so that the other student is folding and being convincing and the other is being a skeptic. If you have an odd number of students in the class there are 2 options:

One group works as a group of 3 with 2 students defending for one skeptic each time. Don't have 2 skeptics and one defending as our trials showed that this was a lot of pressure for the student defending. If 2 are presenting solutions the skeptic should ask each one to justify the steps. Another option is for the teacher to work in a pair with one of the students.

It doesn't matter how many of the questions the students complete, what matters is that they are experiencing the need to be convincing, and to justify and reason.

When I taught the paper folding activity I chose to introduce the dot card number talk part way through the lesson; this interrupted the students paper folding but I liked doing this as it gave the students something different to think about for a while. You may prefer to do the dot card activity before or after the paper folding task.

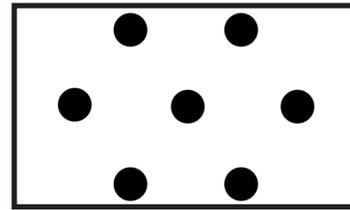


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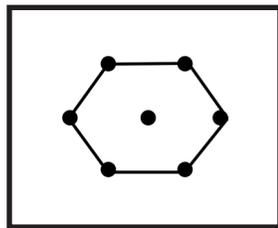
Activity: Dot Card

A dot card number talk is a really nice activity that people of all ages enjoy. It is a short but powerful teaching activity and it shows students:

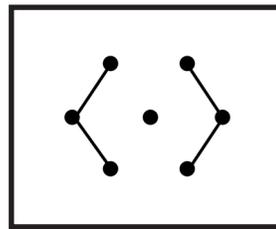
- the creativity in math
- the visual nature of math and
- the many different ways people see math.



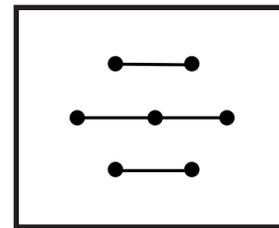
This gives teachers a chance to honor the fact that we all see math differently and that the different ways students see math should be respected. In this 6 minute video, <https://www.youcubed.org/jo-dot-card-number-talk/>, you can see how I used a dot card number talk. I could have added numbers to the students' methods, such as:



$$6 + 1$$



$$3 + 3 + 1$$



$$2 + 3 + 2$$

You can choose whether you want to include the numbers or not. For more detail on teaching a dot card number talk or a regular number talk, see <https://www.youcubed.org/category/teaching-ideas/number-sense/>, Humphreys and Parker (2015), Parrish (2014).

References:

Boaler, J. & Humphreys, C. (2005) *Connecting Mathematical Ideas: Middle School Cases of Teaching & Learning*. Heinemann: Portsmouth.

Humphreys, C. & Parker, R. (2015). *Making Number Talks Matter: Developing Mathematical Practices and Deepening Understanding, Grades 4-10*. Portland, ME: Stenhouse.

Parish, S. (2014). *Number Talks: Helping Children Build Mental Math and Computation Strategies, Grades K-5, Updated with Common Core Connections*. Math Solutions.



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Activity	Time	Description/Prompt	Materials
Day 3 Video: Speed	3 min	Video https://www.youcubed.org/wim-day-3/	
		Possible video discussion	
Paper Folding	35 min	<ol style="list-style-type: none"> 1. Ask students to make a square out of an 8.5 x 11 paper or provide a square sheet of paper for them 2. Ask students to complete #1. Model being a skeptic while one student proves their folded square represents $\frac{1}{4}$ of the area of the square 3. Ask students to continue with 2 - 5 of the paper folding activity and switch roles as the convincer and the skeptic. 	<ul style="list-style-type: none"> • Paper, pencil/pen • Paper Folding handout, page 5 • One square piece of paper for each student. 8.5 x 8.5 is a good size since this is made from a piece of 8.5 x 11 piece of paper
Dot Card Number Talk	5 min	<ol style="list-style-type: none"> 1. Show the dot card visual to students. Put it away before they have time to count and ask them how many dots they saw and how they saw them. See this video for more detail. https://www.youcubed.org/jo-dot-card-number-talk/ 2. Draw as many examples of student representations as possible 	<ul style="list-style-type: none"> • 1 copy of the dot card visual for display, page 6
Closing	5 min	Review the key concepts: Math should never be associated with speed. Students should always pause for reflection and take time to make sense of the math they are learning.	



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Adapted from Driscoll, 2007

Work with a partner. Take turns being the skeptic or the convincer. When you are the convincer your job is to be convincing! Give reasons for all of your statements. Skeptics must be skeptical! Don't be easily convinced. Require reasons and justifications that make sense to you.

For each of the problems below one person should make the shape and then be convincing. Your partner is the skeptic. When you move to the next question switch roles.

Start with a square sheet of paper and make folds to construct a new shape. Then, explain how you know the shape you constructed has the specified area.

1. Construct a square with exactly $\frac{1}{4}$ the area of the original square. Convince your partner that it is a square and has $\frac{1}{4}$ of the area.
2. Construct a triangle with exactly $\frac{1}{4}$ the area of the original square. Convince your partner that it has $\frac{1}{4}$ of the area.
3. Construct another triangle, also with $\frac{1}{4}$ the area, that is not congruent to the first one you constructed. Convince your partner that it has $\frac{1}{4}$ of the area.
4. Construct a square with exactly $\frac{1}{2}$ the area of the original square. Convince your partner that it is a square and has $\frac{1}{2}$ of the area.
5. Construct another square, also with $\frac{1}{2}$ the area, that is oriented differently from the one you constructed in 4. Convince your partner that it has $\frac{1}{2}$ of the area.

