PARCC MODEL CONTENT FRAMEWORKS

MATHEMATICS

ALGEBRA I

Version 3.0

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Algebra I Overview

Numerals in parentheses designate individual content standards that are eligible for assessment in whole or in part. Underlined numerals (e.g., 1) indicate standards eligible for assessment on two or more end-of-course assessments. For more information, see Tables 1 and 2. Course emphases are indicated by: ■ Major Content; ☐ Supporting Content; ○ Additional Content. Not all CCSSM content standards in a listed domain or cluster are assessed.

The Real Number System (N-RN)

○ Use properties of rational and irrational numbers (3)

Quantities★(N-Q)

☐ Reason quantitatively and use units to solve problems (1, 2, 3)

Seeing Structure in Expressions (A-SSE)

■ Interpret the structure of expressions (1, 2)

☐ Write expressions in equivalent forms to solve problems (3)

Arithmetic with Polynomials and Rational Expressions (A-APR)

■ Perform arithmetic operations on polynomials (1)

☐ Understand the relationship between zeros and factors of polynomials (3)

Creating Equations★ (A-CED)

■ Create equations that describe numbers or relationships (1, 2, 3, 4)

Reasoning with Equations and Inequalities (A-REI)

■ Understand solving equations as a process of reasoning and explain the reasoning (1)

■ Solve equations and inequalities in one variable (3, 4)

○ Solve systems of equations (5, 6)

■ Represent and solve equations and inequalities graphically (10, 11, 12)

Interpreting Functions (F-IF)

■ Understand the concept of a function and use function notation (1, 2, 3)

■ Interpret functions that arise in applications in terms of the context (4, 5, 6)

☐ Analyze functions using different representations (7, 8, 9)

Mathematical Practices

1. Make sense of problems and persevere in solving them.

2. Reason abstractly and quantitatively.

3. Construct viable arguments and critique the reasoning of others.

4. Model with mathematics.

5. Use appropriate tools strategically.

6. Attend to precision.

7. Look for and make use of structure.

8. Look for and express regularity in repeated reasoning.

Building Functions (F-BF)

- ◻ Build a function that models a relationship between two quantities (1)
- ○ Build new functions from existing functions (3)

Linear, Quadratic, and Exponential Models★ (F-LE)

- ◻ Construct and compare linear, quadratic, and exponential models and solve problems (1, 2, 3)
- ◻ Interpret expressions for functions in terms of the situation they model (5)

Interpreting categorical and quantitative data (S-ID)

- ○ Summarize, represent, and interpret data on a single count or measurement variable (1, 2, 3)
- ◻ Summarize, represent, and interpret data on two categorical and quantitative variables (5, 6)
- ◼ Interpret linear models (7, 8, 9)

Examples of Key Advances from Grades K–8

- Having already extended arithmetic from whole numbers to fractions (grades 4-6) and from fractions to rational numbers (grade 7), students in grade 8 encountered particular irrational numbers such as $\sqrt{5}$ or π. In Algebra I, students will begin to understand the real number *system*. For more on the extension of number systems, see page 58 of the standards.

- Students in middle grades worked with measurement units, including units obtained by multiplying and dividing quantities. In Algebra I, students apply these skills in a more sophisticated fashion to solve problems in which reasoning about units adds insight (N-Q).

- Themes beginning in middle school algebra continue and deepen during high school. As early as grades 6 and 7, students began to use the properties of operations to generate equivalent expressions (6.EE.3, 7.EE.1). By grade 7, they began to recognize that rewriting expressions in different forms could be useful in problem solving (7.EE.2). In Algebra I, these aspects of algebra carry forward as students continue to use properties of operations to rewrite expressions, gaining fluency and engaging in what has been called "mindful manipulation."[1]

- Students in grade 8 extended their prior understanding of proportional relationships to begin working with functions, with an emphasis on linear functions. In Algebra I, students will master linear and quadratic functions. Students encounter other kinds of functions to ensure that general principles are perceived in generality, as well as to enrich the range of quantitative relationships considered in problems.

- Students in grade 8 connected their knowledge about proportional relationships, lines, and linear equations (8.EE.5, 6). In Algebra I, students solidify their understanding of the analytic geometry of lines. They understand that in the Cartesian coordinate plane:

  o The graph of any linear equation in two variables is a line.

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[1] See, for example, "Mindful Manipulation," in *Focus in High School Mathematics: Reasoning and Sense Making* (National Council of Teachers of Mathematics, 2009).

  o Any line is the graph of a linear equation in two variables.

- As students acquire mathematical tools from their study of algebra and functions, they apply these tools in statistical contexts (e.g., S-ID.6). In a modeling context, they might informally fit a quadratic function to a set of data, graphing the data and the model function on the same coordinate axes. They also draw on skills they first learned in middle school to apply basic statistics and simple probability in a modeling context. For example, they might estimate a measure of center or variation and use it as an input for a rough calculation.

- Algebra I techniques open a huge variety of word problems that can be solved that were previously inaccessible or very complex in grades K-8. This expands problem solving from grades K-8 dramatically.

Discussion of Mathematical Practices in Relation to Course Content

Two overarching practices relevant to Algebra I are:

- **Make sense of problems and persevere in solving them** (MP.1).

- **Model with mathematics** (MP.4).

Indeed, other mathematical practices in Algebra I might be seen as contributing specific elements of these two. The intent of the following set is not to decompose the above mathematical practices into component parts but rather to show how the mathematical practices work together.

- **Reason abstractly and quantitatively** (MP.2). This practice standard refers to one of the hallmarks of algebraic reasoning, the process of decontextualization and contextualization. Much of elementary algebra involves creating abstract algebraic models of problems (A-CED, F-BF) and then transforming the models via algebraic calculations (A-SSE, A-APR, F-IF) to reveal properties of the problems.

- **Use appropriate tools strategically** (MP.5). Spreadsheets, a function modeling language, graphing tools, and many other technologies can be used strategically to gain understanding of the ideas expressed by individual content standards and to model with mathematics.

- **Attend to precision** (MP.6). In algebra, the habit of using precise language is not only a mechanism for effective communication but also a tool for understanding and solving problems. Describing an idea precisely (A-CED, A-REI) helps students understand the idea in new ways.

- **Look for and make use of structure** (MP.7). For example, writing $49x^2 + 35x + 6$ as $(7x)^2 + 5(7x) + 6$, a practice many teachers refer to as "chunking,'' highlights the structural similarity between this expression and z^2 + 5z + 6, leading to a factorization of the original: ((7x) + 3) ((7x) + 2) (A-SSE, A-APR).

- **Look for and express regularity in repeated reasoning** (MP.8). Creating equations or functions to model situations is harder for many students than working with the resulting expressions. An effective way to help students develop the skill of describing general relationships is to work through several specific examples and then express what they are doing with algebraic symbolism (A-CED). For example, when comparing two different text messaging plans, many students who can compute the cost for a given number of minutes have a hard time writing

general formulas that express the cost of each plan for *any* number of minutes. Constructing these formulas can be facilitated by methodically calculating the cost for several different input values and then expressing the steps in the calculation, first in words and then in algebraic symbols. Once such expressions are obtained, students can find the break-even point for the two plans, graph the total cost against the number of messages sent, and make a complete analysis of the two plans.

Fluency Recommendations

A/G Algebra I students become fluent in solving characteristic problems involving the analytic geometry of lines, such as writing down the equation of a line given a point and a slope. Such fluency can support them in solving less routine mathematical problems involving linearity, as well as in modeling linear phenomena (including modeling using systems of linear inequalities in two variables).

A-APR.1 Fluency in adding, subtracting, and multiplying polynomials supports students throughout their work in algebra, as well as in their symbolic work with functions. Manipulation can be more mindful when it is fluent.

A-SSE.1b Fluency in transforming expressions and chunking (seeing parts of an expression as a single object) is essential in factoring, completing the square, and other mindful algebraic calculations.

Pathway Summary Table: AI – G – AII Pathway

Table 1. This table summarizes what will be assessed on PARCC end-of-course assessments. A dot indicates that the standard is assessed in the indicated course. Shaded standards are addressed in more than one course. Algebra I and II are adjacent so as to make the shading continuous, despite being generally taught a year apart.

Category	CCSSM Standard	A I	A II	G
Number & Quantity	N-RN.1		•	
	N-RN.2		•	
	N-RN.3	•		
	N-Q.1	•		
	N-Q.2	•	•	
	N-Q.3	•		
	N-CN.1		•	
	N-CN.2		•	
	N-CN.7		•	
Algebra	A-SSE.1	•		
	A-SSE.2	•	•	
	A-SSE.3a	•		
	A-SSE.3b	•		
	A-SSE.3c	•	•	
	A-SSE.4		•	
	A-APR.1	•		
	A-APR.2		•	
	A-APR.3	•	•	
	A-APR.4		•	
	A-APR.6		•	
	A-CED.1	•	•	
	A-CED.2	•		
	A-CED.3	•		
	A-CED.4	•		
	A-REI.1	•	•	
	A-REI.2		•	
	A-REI.3	•		
	A-REI.4a	•		
	A-REI.4b	•	•	
	A-REI.5	•		
	A-REI.6	•	•	
	A-REI.7		•	
	A-REI.10	•		
	A-REI.11	•	•	
	A-REI.12	•		
Functions	F-IF.1	•		
	F-IF.2	•		
	F-IF.3	•	•	
	F-IF.4	•	•	
	F-IF.5	•		
	F-IF.6	•	•	
	F-IF.7a	•		
	F-IF.7b	•		
	F-IF.7c		•	
	F-IF.7e		•	
	F-IF.8a	•		
	F-IF.8b		•	
	F-IF.9	•	•	
	F-BF.1a	•	•	
	F-BF.1b		•	
	F-BF.2		•	
	F-BF.3	•	•	
	F-BF.4a		•	
	F-LE.1	•		
	F-LE.2	•	•	
	F-LE.3	•		
	F-LE.4		•	
	F-LE.5	•	•	
	F-TF.1		•	
	F-TF.2		•	
	F-TF.5		•	
	F-TF.8		•	

Category	CCSSM Standard	A I	A II	G
Geometry	G-CO.1			•
	G-CO.2			•
	G-CO.3			•
	G-CO.4			•
	G-CO.5			•
	G-CO.6			•
	G-CO.7			•
	G-CO.8			•
	G-CO.9			•
	G-CO.10			•
	G-CO.11			•
	G-CO.12			•
	G-CO.13			•
	G-SRT.1			•
	G-SRT.2			•
	G-SRT.3			•
	G-SRT.4			•
	G-SRT.5			•
	G-SRT.6			•
	G-SRT.7			•
	G-SRT.8			•
	G-C.1			•
	G-C.2			•
	G-C.3			•
	G-C.5			•
	G-GPE.1			•
	G-GPE.2		•	
	G-GPE.4			•
	G-GPE.5			•
	G-GPE.6			•
	G-GPE.7			•
	G-GMD.1			•
	G-GMD.3			•
	G-GMD.4			•
	G-MG.1			•
	G-MG.2			•
	G-MG.3			•
Statistics	S-ID.1	•		
	S-ID.2	•		
	S-ID.3	•		
	S-ID.4		•	
	S-ID.5	•		
	S-ID.6a	•	•	
	S-ID.6b	•		
	S-ID.6c	•		
	S-ID.7	•		
	S-ID.8	•		
	S-ID.9	•		
	S-IC.1		•	
	S-IC.2		•	
	S-IC.3		•	
	S-IC.4		•	
	S-IC.5		•	
	S-IC.6		•	
	S-CP.1		•	
	S-CP.2		•	
	S-CP.3		•	
	S-CP.4		•	
	S-CP.5		•	
	S-CP.6		•	
	S-CP.7		•	

Assessment Limits for Standards Assessed on More Than One End-of-Course Test: AI-G-AII Pathway

Table 2. This draft table shows assessment limits for standards assessed on more than one end-of-course test. (These "cross-cutting" standards are visible as shaded cells in Table 1.)

CCSSM Cluster	CCSSM Key	CCSSM Standard	Algebra I Assessment Limits and Clarifications	Algebra II Assessment Limits and Clarifications
Reason quantitatively and use units to solve problems	N-Q.2	Define appropriate quantities for the purpose of descriptive modeling.	This standard will be assessed in Algebra I by ensuring that some modeling tasks (involving Algebra I content or securely held content from grades 6-8) require the student to create a quantity of interest in the situation being described (i.e., a quantity of interest is not selected for the student by the task). For example, in a situation involving data, the student might autonomously decide that a measure of center is a key variable in a situation, and then choose to work with the mean.	This standard will be assessed in Algebra II by ensuring that some modeling tasks (involving Algebra II content or securely held content from previous grades and courses) require the student to create a quantity of interest in the situation being described (i.e., this is not provided in the task). For example, in a situation involving periodic phenomena, the student might autonomously decide that amplitude is a key variable in a situation, and then choose to work with peak amplitude.
Interpret the structure of expressions	A-SSE.2	Use the structure of an expression to identify ways to rewrite it. *For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.*	i) Tasks are limited to numerical expressions and polynomial expressions in one variable. ii) Examples: Recognize $53^2 - 47^2$ as a difference of squares and see an opportunity to rewrite it in the easier-to-evaluate form $(53+47)(53-47)$. See an opportunity to rewrite $a^2 + 9a + 14$ as $(a+7)(a+2)$.	i) Tasks are limited to polynomial, rational, or exponential expressions. ii) Examples: see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$. In the equation $x^2 + 2x + 1 + y^2 = 9$, see an opportunity to rewrite the first three terms as $(x+1)^2$, thus recognizing the equation of a circle with radius 3 and center $(-1, 0)$. See $(x^2 + 4)/(x^2 + 3)$ as $(\,(x^2+3) + 1\,)/(x^2+3)$, thus recognizing an opportunity to write it as $1 + 1/(x^2 + 3)$.
Write expressions in equivalent forms to solve problems	A-SSE.3c	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.↔ (c) Use the properties of exponents to transform expressions for exponential functions. *For example the expression 1.15^t can be rewritten as $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.*	i) Tasks have a real-world context. As described in the standard, there is an interplay between the mathematical structure of the expression and the structure of the situation such that choosing and producing an equivalent form of the expression reveals something about the situation. ii) Tasks are limited to exponential expressions with integer exponents.	i) Tasks have a real-world context. As described in the standard, there is an interplay between the mathematical structure of the expression and the structure of the situation such that choosing and producing an equivalent form of the expression reveals something about the situation. ii) Tasks are limited to exponential expressions with rational or real exponents.
Understand the relationship between zeros and factors of polynomials	A-APR.3	Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.	i) Tasks are limited to quadratic and cubic polynomials in which linear and quadratic factors are available. For example, find the zeros of $(x - 2)(x^2 - 9)$.	i) Tasks include quadratic, cubic, and quartic polynomials and polynomials for which factors are not provided. For example, find the zeros of $(x^2 - 1)(x^2 + 1)$

CCSSM Cluster	CCSSM Key	CCSSM Standard	Algebra I Assessment Limits and Clarifications	Algebra II Assessment Limits and Clarifications
Create equations that describe numbers or relationships	A-CED.1	Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*	i) Tasks are limited to linear, quadratic, or exponential equations with integer exponents.	i) Tasks are limited to exponential equations with rational or real exponents and rational functions. ii) Tasks have a real-world context.
Understand solving equations as a process of reasoning and explain the reasoning	A-REI.1	Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.	i) Tasks are limited to quadratic equations.	i) Tasks are limited to simple rational or radical equations.
Solve equations and inequalities in one variable	A-REI.4b	Solve quadratic equations in one variable. b) Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.	i) Tasks do not require students to write solutions for quadratic equations that have roots with nonzero imaginary parts. However, tasks can require the student to recognize cases in which a quadratic equation has no real solutions. *Note, solving a quadratic equation by factoring relies on the connection between zeros and factors of polynomials (cluster A-APR.B). Cluster A-APR.B is formally assessed in A2.*	i) In the case of equations that have roots with nonzero imaginary parts, students write the solutions as $a \pm bi$ for real numbers a and b.
Solve systems of equations	A-REI.6	Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.	i) Tasks have a real-world context. ii) Tasks have hallmarks of modeling as a mathematical practice (less defined tasks, more of the modeling cycle, etc.).	i) Tasks are limited to 3x3 systems.
Represent and solve equations and inequalities graphically	A-REI.11	Explain why the x-coordinates of the points where the graphs of the equations y=f(x) and y=g(x) intersect are the solutions of the equation f(x) =g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. ★	i) Tasks that assess conceptual understanding of the indicated concept may involve any of the function types mentioned in the standard except exponential and logarithmic functions. ii) Finding the solutions approximately is limited to cases where f(x) and g(x) are polynomial functions.	i) Tasks may involve any of the function types mentioned in the standard.
Understand the concept of a function and use function notation	F-IF.3	Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1) for n ≥ 1.*	i) This standard is part of the Major work in Algebra I and will be assessed accordingly.	i) This standard is Supporting work in Algebra II. This standard should support the Major work in F-BF.2 for coherence.

CCSSM Cluster	CCSSM Key	CCSSM Standard	Algebra I Assessment Limits and Clarifications	Algebra II Assessment Limits and Clarifications
Interpret functions that arise in applications in terms of a context	F-IF.4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.* ★	i) Tasks have a real-world context. ii) Tasks are limited to linear functions, quadratic functions, square root functions, cube root functions, piecewise-defined functions (including step functions and absolute value functions), and exponential functions with domains in the integers. *Compare note (ii) with standard F-IF.7.* *The function types listed here are the same as those listed in the Algebra I column for standards F-IF.6 and F-IF.9.*	i) Tasks have a real-world context ii) Tasks may involve polynomial, exponential, logarithmic, and trigonometric functions. *Compare note (ii) with standard F-IF.7.* *The function types listed here are the same as those listed in the Algebra II column for standards F-IF.6 and F-IF.9.*
Interpret functions that arise in applications in terms of a context	F-IF.6	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. ★	i) Tasks have a real-world context. ii) Tasks are limited to linear functions, quadratic functions, square root functions, cube root functions, piecewise-defined functions (including step functions and absolute value functions), and exponential functions with domains in the integers. *The function types listed here are the same as those listed in the Algebra I column for standards F-IF.4 and F-IF.9.*	i) Tasks have a real-world context. ii) Tasks may involve polynomial, exponential, logarithmic, and trigonometric functions. *The function types listed here are the same as those listed in the Algebra II column for standards F-IF.4 and F-IF.9.*
Analyze functions using different representations	F-IF.9	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions.) *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*	i) Tasks are limited to linear functions, quadratic functions, square root functions, cube root functions, piecewise-defined functions (including step functions and absolute value functions), and exponential functions with domains in the integers. *The function types listed here are the same as those listed in the Algebra I column for standards F-IF.4 and F-IF.6.*	i) Tasks may involve polynomial, exponential, logarithmic, and trigonometric functions. *The function types listed here are the same as those listed in the Algebra II column for standards F-IF.4 and F-IF.6.*
Build a function that models a relationship between two quantities	F-BF.1a	Write a function that describes a relationship between two quantities.★ a) Determine an explicit expression, a recursive process, or steps for calculation from a context.	i) Tasks have a real-world context. ii) Tasks are limited to linear functions, quadratic functions, and exponential functions with domains in the integers.	i) Tasks have a real-world context ii) Tasks may involve linear functions, quadratic functions, and exponential functions.
Build new functions from	F-BF.3	Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x+k) for specific values of k (both positive and negative); find the value of	i) Identifying the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x+k) for specific values of k (both positive and negative) is limited	i) Tasks may involve polynomial, exponential, logarithmic, and trigonometric functions ii) Tasks may involve recognizing even and odd

CCSSM Cluster	CCSSM Key	CCSSM Standard	Algebra I Assessment Limits and Clarifications	Algebra II Assessment Limits and Clarifications
existing functions		*k* given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*	to linear and quadratic functions. ii) Experimenting with cases and illustrating an explanation of the effects on the graph using technology is limited to linear functions, quadratic functions, square root functions, cube root functions, piecewise-defined functions (including step functions and absolute value functions), and exponential functions with domains in the integers. iii) Tasks do not involve recognizing even and odd functions. *The function types listed in note (ii) are the same as those listed in the Algebra I column for standards F-IF.4, F-IF.6, and F-IF.9.*	functions. *The function types listed in note (i) are the same as those listed in the Algebra II column for standards F-IF.4, F-IF.6, and F-IF.9.*
Construct and compare linear, quadratic, and exponential models and solve problems	F-LE.2	Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).	i) Tasks are limited to constructing linear and exponential functions in simple context (not multi-step).	i) Tasks will include solving multi-step problems by constructing linear and exponential functions.
Interpret expressions for functions in terms of the situation they model	F-LE.5	Interpret the parameters in a linear or exponential function in terms of a context.	i) Tasks have a real-world context. ii) Exponential functions are limited to those with domains in the integers.	i) Tasks have a real-world context. ii) Tasks are limited to exponential functions with domains not in the integers.
Summarize, represent, and interpret data on two categorical and quantitative variables	S-ID.6a	Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. a) Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.*	i) Tasks have a real-world context. ii) Exponential functions are limited to those with domains in the integers.	i) Tasks have a real-world context. ii) Tasks are limited to exponential functions with domains not in the integers and trigonometric functions.