Practice Exercises: Normal Distributions

1. The distribution of blood cholesterol levels in 14-year-old boys is roughly normal; the mean is 165 milligrams of cholesterol per deciliter of blood and the standard deviation is 30.

- a) Label the horizontal axis appropriately on the Normal curve at right.
- b) What proportion of 14-year-old boys have level between 120 and 200? Shade the appropriate area in the figure, and then determine the proportion.



- c) What proportion have level *over* 20? (The table can help with this one, but you'll need to use general principles to answer.)
- d) [5] What level separates the boys who have level in the highest 70% from those in the lowest 30%?



2. Consider SAT scores for 2009 high school graduates. The scores are approximately Normally distributed.

- a) [4] What is the approximate percent of graduates scoring within one standard deviation of the mean score?
- b) [4] Jeff's Z score is -1.75. What does this tell us about Jeff's SAT score?

Jeff's score is \_\_\_\_\_\_ standard deviations \_\_\_\_\_\_ the mean.

c) [2] All the SAT scores are converted to *Z* scores. What are the mean and standard deviation of these *Z* scores? (Visualize the *Z* scale beneath the SAT scale.)

Mean of Z scores = \_\_\_\_\_

S D of *Z* scores = \_\_\_\_\_

3. Lifetimes of a certain brand of tires is approximately normally distributed with mean 42,500 miles and standard deviation 3,200 miles

- a) Fill in appropriate axis label at the tick marks on the graph at right. Identify the areas associated with the 68-95-99.7 rule.
- b) In what range do the middle 95% of tire lifetimes lie?
  - \_\_\_\_\_ to \_\_\_\_\_
- c) What proportion of tires last over 45,000 miles?

nearest 0.0001

d) What is the median lifetime?

e) How far do the longest-lasting 4% of these tires roll? (What percentile are you finding here?)

miles

f) Customers get a refund if a tire has unacceptable tread that puts it in the bottom 12% of all tires. At what mileage is a customer no longer entitled to a refund?(What percentile are you finding here?)



miles

## Solutions

1. b) Zhigh = (200 - 165)/30 = 1.17. Zlow = (120 - 165)/30 = -1.50. The probabilities are 0.8783 and 0.0.0668 respectively. The answer is 0.8783 - 0.0668 = 0.8115.

c) The Z score is (20 - 165)/30 = -4.83. Virtually ALL  $\rightarrow 1.0000 = 100\%$ . (Actually: 0.99999933 = 99.999933%.)

d) For the 30<sup>th</sup> percentile the Z score is -0.5244. So the level is 0.5244 standard deviations below the mean: 165 - 0.5244(30) = 149.27.

2. a) <u>68</u>%. b) "...<u>1.75</u> standard deviations <u>below</u>..." c) MEAN =  $\underline{0}$ ; SD =  $\underline{1}$ .

3. b) from about 36,100 (2 sds below the mean) to 48,900 (2 sds above) miles. c) Z = 0.78125; 1 - 0.7827 = 0.2173. d) 42,500 - the distribution is symmetric, so mean = median. e) The 96<sup>th</sup> percentile has Z = 1.7507 which gives 48,102.2 miles. f) The 12<sup>th</sup> percentile has Z = -1.175 equivalent to 38,740 miles.

## **More Exercises**

- 1. The angiogram is a standard diagnostic test used in clinical medicine to detect stroke in patients. This test has some risks for the patient, and several noninvasive techniques have been developed that are hoped to be as effective as the angiogram. One such method utilizes that measurement of cerebral blood flow (CBF) in the brain, since stroke patients tend to have lower levels of CBF than normal. Among healthy people, CBF is normally distributed with mean 75 and standard deviation 17. Patients are classified as being at risk for stroke if their CBF is below 40. What proportion of normal patients will be mistakenly classified as being at risk for stroke?
- 2. Maple tree diameters in a forest area are normally distributed with mean 10 inches and standard deviation 2.2 inches. Find the proportion of trees having a diameter greater than 15 inches.
- 3. Our subjects are 35-44-year-old males whose blood pressures are normally distributed with mean 80 and standard deviation 12 A borderline hypertensive is defined as a person whose diastolic blood pressure is between 90 and 95 mm Hg inclusive; what proportion of subjects are borderline hypertensive? A hypertensive is a person whose diastolic blood pressure is above 95 mm Hg; what proportion of subjects are hypertensive?
- 4. White blood cell (WBC) count per cubic millimeter of whole blood has approximately the Normal distribution with mean 7500 and standard deviation 1750. The lowest 2% of all WBC counts are defined to be probable risks. How low must one's WBC count be to fall in the at-risk group?
- 5. The resting heart rate for healthy adult horses averages 46 beats per minute with a standard deviation of 8 beats per minute. A horse whose resting heart rate is in the upper 10% of the distribution of heart rates may have a secondary infection or illness that needs to be treated. How fast must a healthy horse's heart be beating to fall into this at-risk group?

## Solutions

- 1. 0.0198
- 2. 0.0115
- 3. 0.0967, 0.1056
- 4. 3905.94 (3906 is fine)
- 5. 56.25