

Math Test – Calculator Answer Explanations

Question 1

Tyra subscribes to an online gaming service that charges a monthly fee of \$5.00 and \$0.25 per hour for time spent playing premium games. Which of the following functions gives Tyra's cost, in dollars, for a month in which she spends x hours playing premium games?

- A) $C(x) = 5.25x$
- B) $C(x) = 5x + 0.25$
- C) $C(x) = 5 + 0.25x$
- D) $C(x) = 5 + 25x$

Item Difficulty: Easy

Content: Heart of Algebra

Correct Answer: C

Choice C is the correct answer. Tyra pays \$0.25 per hour for time spent playing premium games, so for the month in which she spends x hours playing premium games, she pays $0.25x$ dollars for playing the premium games. She also pays an additional \$5 monthly fee. Therefore, Tyra's cost, in dollars, for the month in which she spends x hours playing premium games is given by the function $C(x) = 5 + 0.25x$.

Choice A is incorrect because Tyra is not charged \$5.25 per hour for time playing premium games. Choice B is incorrect because the charge per hour has been interchanged with the monthly fee. Choice D is incorrect because $25x$ is the charge for playing premium games in cents, not in dollars.

Question 2

A grocery store sells a brand of juice in individual bottles and in packs of 6 bottles. On a certain day, the store sold a total of 281 bottles of the brand of juice, of which 29 were sold as individual bottles. Which equation shows the number of packs of bottles, p , sold that day?

A) $p = \frac{281 - 29}{6}$

B) $p = \frac{281 + 29}{6}$

C) $p = \frac{281}{6} - 29$

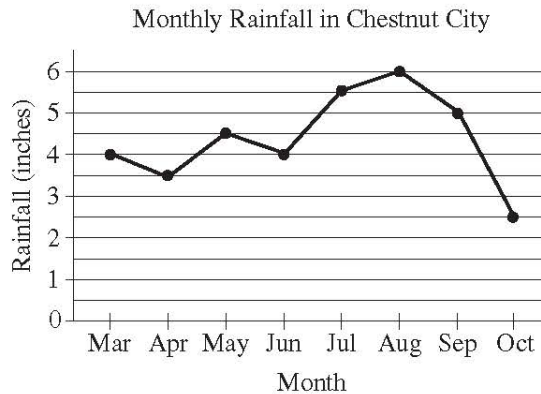
D) $p = \frac{281}{6} + 29$

Item Difficulty: Easy
Content: Heart of Algebra
Correct Answer: A

Choice A is the correct answer. Since the store sold a total of 281 bottles, 29 of which were sold individually, it follows that $281 - 29$ bottles were sold in packs of 6 bottles. Therefore, the number of packs of bottles, p , sold that day in the store is $p = \frac{281 - 29}{6}$.

Choice B is incorrect. Adding the number of bottles sold individually, 29, to the total number of bottles sold, 281, does not give the number of bottles that were sold in packs of 6. Choices C and D are incorrect and could result from dividing all of the bottles into groups of 6 (incorrectly assuming that all 281 bottles of juice were sold in packs of 6), and either subtracting the 29 bottles sold individually from that result, as in choice C, or adding the 29 bottles to that result, as in choice D.

Question 3



The line graph above shows the monthly rainfall from March to October last year in Chestnut City. According to the graph, what was the greatest change (in absolute value) in the monthly rainfall between two consecutive months?

- A) 1.5 inches
- B) 2.0 inches
- C) 2.5 inches
- D) 3.5 inches

Item Difficulty: Medium

Content: Probability and Data Analysis

Correct Answer: C

Choice C is the correct answer. The greatest change (in absolute value) in monthly rainfall could be an increase or a decrease in monthly rainfall. The table below shows the approximate changes in monthly rainfall in Chestnut City last year between each of the two consecutive months.

Consecutive months	Change in monthly rainfall (inches)
March to April	0.5
April to May	1
May to June	0.5
June to July	1.5
July to August	0.5
August to September	1
September to October	2.5

Of the values on the right column, the greatest is from September to October, which is a change of 2.5 inches.

Choices A, B, and D are incorrect because they contain values that either do not represent any of the changes in monthly rainfall between two consecutive months or that are not the greatest change.

Question 4

A rectangle has perimeter P , length ℓ and width w . Which of the following represents ℓ in terms of P and w ?

A) $\ell = P - w$

B) $\ell = \frac{2P - w}{2}$

C) $\ell = \frac{P - 2w}{2}$

D) $\ell = 2P - 2w$

Item Difficulty: Medium

Content: Passport to Advanced Math

Correct Answer: C

Choice C is the correct answer. The perimeter of a rectangle is the sum of the four sides and can be calculated using the formula $P = 2\ell + 2w$, where ℓ is the length and w is the width of the rectangle. Subtracting $2w$ from both sides of the equation gives

$$P - 2w = 2\ell, \text{ and then dividing by } 2 \text{ yields } \ell = \frac{P - 2w}{2}.$$

Choice A is incorrect. This choice does not use the fact that the perimeter of a rectangle is the sum of two length and two widths. Choice B and D are incorrect. In each of these choices, the equation incorrectly doubles the perimeter.

Question 5

Which ordered pair (x, y) satisfies the system of equations shown below?

$$\begin{aligned}2x - y &= 6 \\x + 2y &= -2\end{aligned}$$

- A) $(-6, 2)$
- B) $(-2, 2)$
- C) $(2, -2)$
- D) $(4, 2)$

Item Difficulty: Medium
Content: Heart of Algebra
Correct Answer: C

Choice C is the correct answer. To eliminate y , the first equation in the system can be multiplied by 2 and then the equations can be added as shown below.

$$\begin{array}{r}4x - 2y = 12 \\x + 2y = -2 \\ \hline5x + 0 = 10\end{array}$$

Since the result is $5x = 10$, it follows that $x = 2$. Substituting 2 for x into the equation $x + 2y = -2$ gives $2 + 2y = -2$ and so $y = -2$. Therefore, $(2, -2)$ is the solution to the system given.

Alternatively: Use the substitution method to solve the system. For example, the first equation can be rewritten as $y = 2x - 6$. Substituting $2x - 6$ for y in the second equation gives $x + 2(2x - 6) = -2$, and so $x = 2$. Finally, substituting 2 for x in $y = 2x - 6$ gives $y = -2$, leading to the same solution of the system, namely $(2, -2)$.

Choice B is incorrect. The value for x and the value for y have been reversed in the ordered pair. Choices A and D are incorrect. The ordered pair in each of these choices does not satisfy at least one of the equations in the system. For example, the ordered pair $(4, 2)$ does not satisfy the equation $x + 2y = -2$, since $4 + 2(2) \neq -2$.

Question 6

A soda company is filling bottles of soda from a tank that contains 500 gallons of soda. At most, how many 20-ounce bottles can be filled from the tank? (1 gallon = 128 ounces)

- A) 25
- B) 78
- C) 2,560
- D) 3,200

Item Difficulty: Easy

Content: Probability and Data Analysis

Correct Answer: D

Choice D is the correct answer. Since 1 gallon equals 128 ounces, 500 gallons equal $(500)(128) = 64,000$ ounces. Therefore, the maximum number of 20-ounce bottles that can be filled with the soda from the tank is $\frac{64,000}{20} = 3,200$.

Choice A is incorrect and could result from dividing 500 (the number of gallons contained in the tank) by 20 (the capacity of one bottle, in ounces). The gallons need to be converted into ounces first, and then the result can be divided by 20. Choices B and C are incorrect because they do not give the maximum number of 20-ounce bottles that can be filled from the soda in the tank.

Question 7

A car traveled at an average speed of 80 miles per hour for 3 hours and consumed fuel at a rate of 34 miles per gallon. Approximately how many gallons of fuel did the car use for the entire 3-hour trip?

- A) 2
- B) 3
- C) 6
- D) 7

Item Difficulty: Medium

Content: Probability and Data Analysis

Correct Answer: D

Choice D is the correct answer. Since the car traveled at an average speed of 80 miles per hour, the distance the car traveled during 3 hours is $(80)(3) = 240$ miles.

The car consumed fuel at a rate of 34 miles per gallon, so the car used $\frac{240}{34}$ gallons of fuel, which is approximately 7 gallons of fuel.

Choices A, B, and C are incorrect. For each of these choices, the amount of fuel is not enough to travel the entire 240 miles.

Question 8

What is the slope of the line in the xy -plane that passes through the points

$$\left(-\frac{5}{2}, 1\right) \text{ and } \left(-\frac{1}{2}, 4\right) ?$$

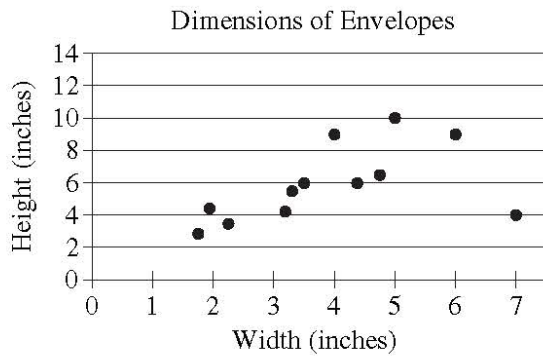
- A) -1
- B) $-\frac{2}{3}$
- C) 1
- D) $\frac{3}{2}$

Item Difficulty: Medium
Content: Heart of Algebra
Correct Answer: D

Choice D is the correct answer. In the xy -plane, the slope m of a line that passes through the points (x_1, y_1) and (x_2, y_2) is the change in y over the change in x (rise over run), which is expressed by the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$. Thus, the slope of the line through the points $\left(-\frac{5}{2}, 1\right)$ and $\left(-\frac{1}{2}, 4\right)$ is $\frac{4 - 1}{-\frac{1}{2} - \left(-\frac{5}{2}\right)}$, which simplifies to $\frac{3}{2}$.

Choices A and C are incorrect because the change in y and the change in x do not have the same magnitude. Choice B is incorrect; the fraction $-\frac{2}{3}$ is the negative reciprocal of the slope of the line through the points $\left(-\frac{5}{2}, 1\right)$ and $\left(-\frac{1}{2}, 4\right)$.

Question 9



The scatterplot above shows the widths and the heights of 12 types of rectangular envelopes. What is the width, in inches, of the envelope represented by the data point that is farthest from the line of best fit (not shown)?

- A) 2
- B) 5
- C) 7
- D) 12

Item Difficulty: Medium

Content: Probability and Data Analysis

Correct Answer: C

Choice C is the correct answer. The data point that is farthest from the line of best fit is located at (7, 4), which means that this point represents a type of envelope that is 7 inches wide and 4 inches high.

Choices A and B are incorrect because none of the data points with width 2 or width 5 is the farthest from the line of best fit. Choice D is incorrect because the scatterplot does not contain any points with width 12 inches.

Question 10

A high school basketball team won exactly 65 percent of the games it played during last season. Which of the following could be the total number of games the team played last season?

- A) 22
- B) 20
- C) 18
- D) 14

Item Difficulty: Medium

Content: Probability and Data Analysis

Correct Answer: B

Choice B is the correct answer. The number of games won by the basketball team must be a whole number. Since 65% is equivalent to $\frac{13}{20}$, it follows that, of the choices given, the total number of games the team played last season can only be 20. Multiplying $\frac{13}{20}$ by each of the other answer choices does not result in a whole number.

Choices A, C, and D are incorrect because 65% of each of the numbers in the choices results in non-whole numbers.

Question 11

$$110x + y = 1,210$$

A coffee shop is running a promotion where a number of free coffee samples are given away each day. The equation above can be used to model the number of free coffee samples, y , that remain to be given away x days after the promotion began. What does it mean that $(11, 0)$ is a solution to this equation?

- A) During the promotion, 11 samples are given away each day.
- B) It takes 11 days during the promotion to see 1,210 customers.
- C) It takes 11 days during the promotion until none of the samples are remaining.
- D) There are 11 samples available at the start of the promotion.

Item Difficulty: Medium

Content: Heart of Algebra

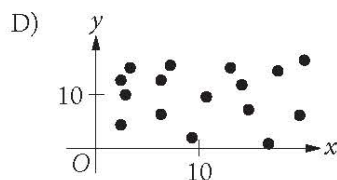
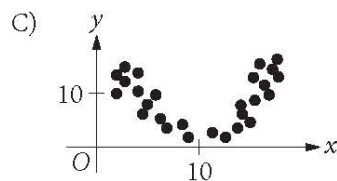
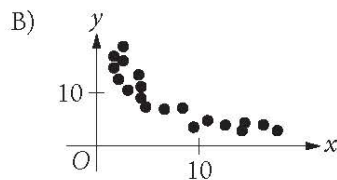
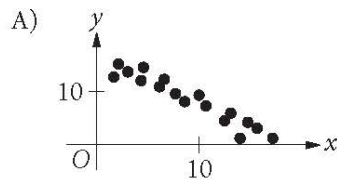
Correct Answer: C

Choice C is the correct answer. Since x represents the number of days after the promotion began and y represents the remaining number of coffee samples, the fact that the ordered pair $(11, 0)$ is a solution to the given equation means that it takes 11 days during the promotion until none of the samples are remaining.

Choice A is incorrect; if 11 samples were given away each day, then the coefficient of x in the equation would be 11. Therefore, this is not the correct interpretation of $(11, 0)$ as a solution to the equation. Choice B is incorrect; the total number of free coffee samples given away during 11 days of the promotion was 1,210. But the number of customers who were in the store during those days need not be 1,210. Choice D is incorrect; according to the given equation, there were 1,210, not 11, samples available at the start of the promotion.

Question 12

Which scatterplot shows a negative association that is not linear? (Note: A negative association between two variables is one in which higher values of one variable correspond to lower values of the other variable, and vice versa.)



Item Difficulty: Medium

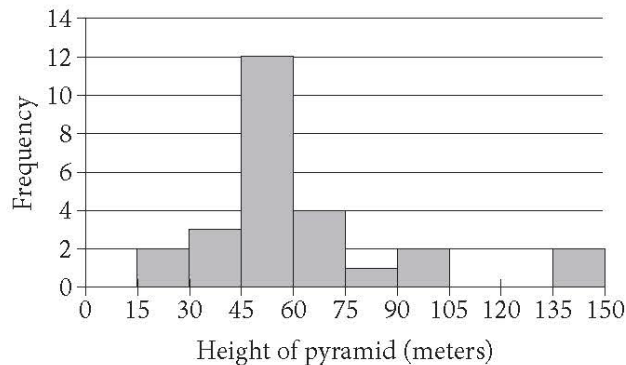
Content: Probability and Data Analysis

Correct Answer: B

Choice B is the correct answer. Of the choices given, only the scatterplots in A and B show a negative association between variables x and y , and of these two associations, the one depicted in choice B is not linear.

Choice A is incorrect. The association depicted in this scatterplot is negative, but it can also be linear. Choice C is incorrect. The association depicted in this scatterplot is not linear. However, for x greater than 10, the association between x and y is positive. Choice D is incorrect. There is no clear association between x and y in this scatterplot.

Question 13



The histogram above shows the distribution of the heights, in meters, of 26 pyramids in Egypt. Which of the following could be the median height of the 26 pyramids represented in the histogram?

- A) 44 meters
- B) 48 meters
- C) 63 meters
- D) 77 meters

Item Difficulty: Medium

Content: Probability and Data Analysis

Correct Answer: B

Choice B is the correct answer. The median of a data set is the middle value when the data points are sorted in either ascending or descending order. When the number of the data points is even, then the median is the mean of the two middle values of the sorted data. Hence, the median height of the 26 pyramids is the mean

of the 13th and 14th tallest pyramids. Since the number of pyramids that are less than 30 meters high is 5 and the number of pyramids that are less than 60 meters high is 17, the median height of the 26 pyramids must be between 45 and 60 meters. Therefore, of the choices given, only 48 meters could be the median height of the 26 pyramids.

Choices A, C, and D are incorrect because the median height of the 26 pyramids cannot be less than 45 meters or greater than 60 meters.

Questions 14-16 refer to the following information.

A survey of 170 randomly selected teenagers aged 14 through 17 in the United States was conducted to gather data on summer employment of teenagers. The data are shown in the table below.

	Have a summer job	Do not have a summer job	Total
Ages 14–15	20	69	89
Ages 16–17	39	42	81
Total	59	111	170

Question 14

Which of the following is closest to the percent of those surveyed who had a summer job?

- A) 22%
- B) 35%
- C) 47%
- D) 53%

Item Difficulty: Medium

Content: Probability and Data Analysis

Correct Answer: B

Choice B is the correct answer. The number of teenagers surveyed in the data is 170. Of those surveyed, a total of 59 teenagers had a summer job; thus, the percent of those teenagers surveyed who had a summer job is $\frac{59}{170} = 0.347$, which rounds to 35%.

Choice A is incorrect. This choice, 22%, is the approximate percent $\left(\frac{20}{89} \approx 0.22\right)$ of teenagers aged 14 to 15 who had summer jobs. But that is not precisely what is

asked in this question. Choices C and D are incorrect and may be the result of calculating relative frequencies that are different from what the problem asks.

Question 15

In 2012 the total population of individuals in the United States who were between 14 and 17 years old (inclusive) was about 17 million. If the survey results are used to estimate information about summer employment of teenagers across the country, which of the following is the best estimate of the total number of individuals between 16 and 17 years old in the United States who had a summer job in 2012?

- A) 8,200,000
- B) 3,900,000
- C) 2,000,000
- D) 390,000

Item Difficulty: Hard

Content: Probability and Data Analysis

Correct Answer: B

Choice B is the correct answer. In 2012, the total population of individuals in the United States who were between 14 and 17 years old (inclusive) was about 17 million, which is 10^5 times the size of the survey sample, 170. Since of those surveyed, 39 teenagers aged 16 to 17 had a summer job, it follows that the best estimate of the total number of individuals aged 16 to 17 in the United States who had a summer job in 2012 was $39 \times 10^5 = 3,900,000$.

Choices A, C, and D are incorrect and are likely the result of either conceptual or calculation errors made.

Question 16

Based on the data, how many times more likely is it for a 14 year old or a 15 year old to NOT have a summer job than it is for a 16 year old or a 17 year old to NOT have a summer job? (Round the answer to the nearest hundredth.)

- A) 0.52 times as likely
- B) 0.65 times as likely
- C) 1.50 times as likely
- D) 1.64 times as likely

Item Difficulty: Hard

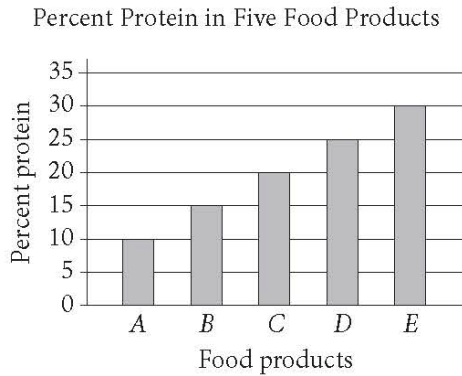
Content: Probability and Data Analysis

Correct Answer: C

Choice C is the correct answer. According to the data shown in the table, 69 out of 89 teenagers aged 14 to 15 did not have summer jobs. So for a 14- or 15-year-old, the likelihood of not having a summer job is $\frac{69}{89}$. And since 42 out of 81 teenagers aged 16 to 17 did not have a summer job, the likelihood that a 15- or 16-year-old not having a summer job is $\frac{42}{81}$. Therefore, a 14- or 15-year-old is $\frac{69}{89} \div \frac{42}{81} = \frac{1,863}{1,246} = 1.49518$, or about 1.50, times more likely to not have a summer job.

Choice A is incorrect. This choice could result from calculating the likelihood that a teenager aged 16 to 17 will not have a summer job $\left(\frac{42}{81}\right)$. Choice B is incorrect. This choice could result from calculating the likelihood that a teenager aged 14 through 17 will not have a summer job is $\left(\frac{111}{170}\right)$. Choice D is incorrect. This choice could result from calculating the ratio of the number of teenagers aged 14 to 15 who do not have a summer job (69) to the number of teenagers aged 16 to 17 who do not have a summer job (42). If the total number of those surveyed in the two different groups were the same, this result would be correct. But the sizes of the two groups are different; therefore, the result obtained is incorrect.

Question 17



The graph above shows the amount of protein supplied by five different food products, *A*, *B*, *C*, *D*, and *E*, as a percentage of their total weights. The costs of 10 grams of products *A*, *B*, *C*, *D*, and *E* are \$2.00, \$2.20, \$2.50, \$4.00, and \$5.00, respectively. Which of the five food products supplies the most protein per dollar?

- A) *A*
- B) *B*
- C) *C*
- D) *E*

Item Difficulty: Medium

Content: Probability and Data Analysis

Correct Answer: C

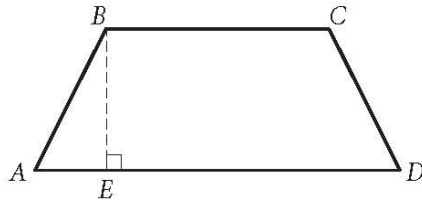
Choice C is the correct answer. The table below organizes the information in the graph and the additional data needed to answer the question.

Food product	Cost of 10 grams of product	Amount of product (in grams)	Percent protein	Amount of protein (in grams)	Protein per dollar (in grams/dollar)
<i>A</i>	\$2.00	10	10%	$0.1(10) = 1$	$\frac{10(0.1)}{2} = 0.5$
<i>B</i>	\$2.20	10	15%	$0.15(10) = 1.5$	$\frac{10(0.15)}{2.2} = 0.68$
<i>C</i>	\$2.50	10	20%	$0.2(10) = 2$	$\frac{10(0.2)}{2.5} = 0.8$
<i>D</i>	\$4.00	10	25%	$0.25(10) = 2.5$	$\frac{10(0.25)}{4} = 0.625$
<i>E</i>	\$5.00	10	30%	$0.3(10) = 3$	$\frac{10(0.3)}{5} = 0.6$

According to the table, food product C provides the most protein per dollar (0.8).

Choices A, B, and D are incorrect. For each choice, the protein per dollar for each of the food products is less than 0.8 grams of protein per dollar.

Question 18



In quadrilateral $ABCD$ above, \overline{BC} is parallel to \overline{AD} , and $AB = CD$. If BC and AD were each doubled and BE was reduced by 50 percent, how would the area of $ABCD$ change?

- A) The area of $ABCD$ would be decreased by 50 percent.
- B) The area of $ABCD$ would be increased by 50 percent.
- C) The area of $ABCD$ would not change.
- D) The area of $ABCD$ would be multiplied by 2.

Item Difficulty: Medium

Content: Passport to Advanced Math

Correct Answer: C

Choice C is the correct answer. Quadrilateral $ABCD$ is a trapezoid, and the formula for the area of a trapezoid is $A = \frac{1}{2}h(b_1 + b_2)$, where b_1 and b_2 are the bases of the trapezoid (BC and AD) and h is the height (BE). If the bases (BC and AD) are each doubled and the height (BE) is reduced by 50%, then the area of the new trapezoid $ABCD$ would be $\frac{1}{2}\left(\frac{h}{2}\right)(2b_1 + 2b_2)$, which after multiplying out becomes $\frac{1}{2}h(b_1 + b_2)$, the same as the area of the original trapezoid. Therefore, the area of the trapezoid would not change.

Choice A is incorrect. This choice does not take into account the changes to the bases, BC and AD . Choice B is incorrect. This choice could result from incorrectly interpreting the impact of doubling the bases on the area of $ABCD$ as a 100% increase and the impact of reducing the height by 50% as a 50% decrease, resulting

in a combined $100\% - 50\% = 50\%$ increase of the area. Choice D is incorrect. This choice does not take into account the change to height, BE .

Question 19

Boyd grows only tomatoes and raspberries in his garden. Last year, he grew 140 pounds of tomatoes and 60 pounds of raspberries. This year, the production, by weight, of tomatoes declined by 20 percent, and the production, by weight, of raspberries declined by 50 percent. By what percentage did the total yield, by weight, of Boyd's garden decline?

- A) 29 percent
- B) 30 percent
- C) 35 percent
- D) 70 percent

Item Difficulty: Hard

Content: Probability and Data Analysis

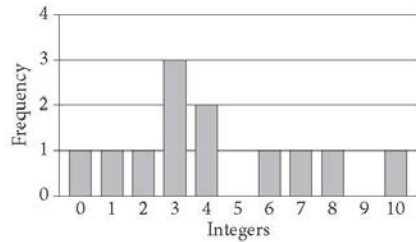
Correct Answer: A

Choice A is the correct answer. Since Boyd's production of tomatoes declined by 20% and the production of raspberries declined by 50% from the previous year, this year, his tomato production was $140 - 0.2(140) = 112$ pounds and his raspberry production was $60 - 0.5(60) = 30$ pounds. The percent decline in the total yield is the decline in the number of pounds of tomatoes and raspberries divided by the original number of pounds of tomatoes and raspberries, which is

$$\frac{28 + 30}{140 + 60} = 0.29 = 29\%.$$

Choice B is incorrect. This choice is close to the answer, but rounding may have erroneously led to this answer. Choice C is incorrect. This choice, 35%, may be a result of calculating the mean of 20% and 50%. Choice D is incorrect. This choice is the approximate percent weight of the tomatoes and raspberries produced this year compared to the last year, but that's not what the problem asks for.

Question 20



The graph above shows the frequency distribution of a list of randomly generated integers between 0 and 10. What is the mean of the list of numbers?

- A) 3.0
- B) 3.5
- C) 4.25
- D) 12.0

Item Difficulty: Medium

Content: Probability and Data Analysis

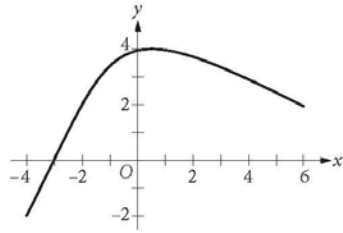
Correct Answer: C

Choice C is the correct answer. There are 12 integers in the list, and some of them are repeated at the frequencies shown in the graph. So the mean of the list of numbers is the sum of the numbers (repeats included) divided by 12. That is

$$\frac{0 + 1 + 2 + 3(3) + 2(4) + 6 + 7 + 8 + 10}{12} = 4.25.$$

Choice A is incorrect; 3 is the mode, not the mean, of the list of numbers. Choice B is incorrect; 3.5 is the median, not the mean, of the list of numbers. Choice D is incorrect; 12 is the total number of the integers in the list.

Question 21



What is the minimum value of the function graphed on the xy -plane above, for $-4 \leq x \leq 6$?

- A) $-\infty$
- B) -4
- C) -2
- D) 1

Item Difficulty: Hard

Content: Passport to Advanced Math

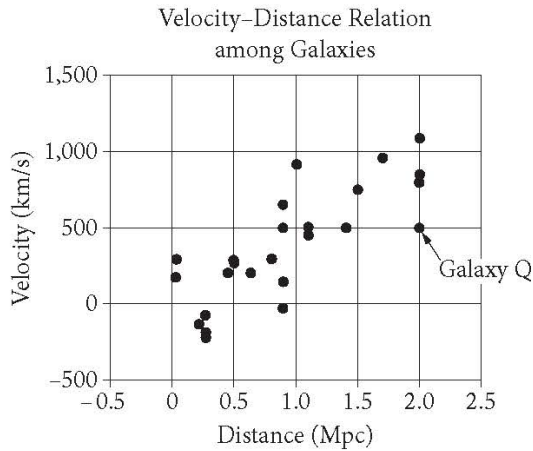
Correct Answer: C

Choice C is the correct answer. The minimum value of a graphed function is the minimum y -value of all the points on the graph. For the graph shown, the minimum is at the left endpoint of the graph, the y -value of which is -2 .

Choice A is incorrect. If the graph would continue indefinitely downward, then the minimum value of the function would be negative infinity. However, the domain of the function is restricted ($-4 \leq x \leq 6$), and the minimum value of the graph occurs at point $(-4, -2)$. Choice B is incorrect; -4 is the x -value of the point on the graph where the minimum value of the function occurs. Choice D is incorrect because there are points on the graph below the x -axis; therefore, the minimum value of the function cannot be positive.

Questions 22-24 refer to the following information.

In 1929, the astronomer Edwin Hubble published the data shown. The graph plots the velocity of galaxies relative to Earth against the distances of galaxies from Earth.



Hubble’s data can be modeled by the equation $v = 500d$, where v is the velocity, in kilometers per second, at which the galaxy is moving away from Earth and d is the distance, in megaparsecs, of the galaxy from Earth. Assume that the relationship is valid for larger distances than are shown in the graph. (A megaparsec (Mpc) is 3.1×10^{19} kilometers.)

Question 22

According to Hubble’s data, how fast, in meters per second, is Galaxy Q moving away from Earth?

- A) 2×10^6 m/s
- B) 5×10^5 m/s
- C) 5×10^2 m/s
- D) 2.5×10^2 m/s

Item Difficulty: Hard

Content: Probability and Data Analysis

Correct Answer: B

Choice B is the correct answer. The coordinates of the data point that represent Galaxy Q on the scatterplot are (2.0, 500), which means that Galaxy Q is at a distance of about 2.0 Mpc from Earth and moves away from Earth at a velocity of approximately 500 km/s. The question asks for the velocity in meters per second; therefore, kilometers (km) need to be converted into meters (m). Since 1 km is

equal to 1,000 m, it follows that Galaxy Q is moving away from Earth at a velocity of $500 \times 1,000$ m/s, or 5×10^5 m/s.

Choices A, C, and D are incorrect and may result from an incorrect interpretation of the coordinates of the point that represents Galaxy Q on the scatterplot or an incorrect conversion of the units.

Question 23

There are four galaxies shown in the graph at approximately 0.9 Mpc from Earth. Which of the following is closest to the range of velocities of these four galaxies, in kilometers per second?

- A) 100
- B) 200
- C) 450
- D) 700

Item Difficulty: Hard

Content: Probability and Data Analysis

Correct Answer: D

Choice D is the correct answer. The velocities, in km/s, of the four galaxies shown in the graph at approximately 0.9 Mpc from Earth are about -50 , $+200$, $+500$, and $+650$. Thus, the range of the four velocities is approximately $650 - (-50) = 700$ km/s.

Choices A, B, and C are incorrect. The range of velocities is the difference between the largest and smallest velocity. Each of these answer choices is too small compared to the real value of the range.

Question 24

Based on the model, what is the velocity, in kilometers per second, of a galaxy that is 15 Mpc from Earth?

- A) 7,500 km/s
- B) 5,000 km/s
- C) 1,100 km/s
- D) 750 km/s

Item Difficulty: Medium

Content: Heart of Algebra

Correct Answer: A

Choice A is the correct answer. The model indicates that the relationship between the velocities of the galaxies, in km/s, and their distance from Earth, in Mpc, is $v = 500d$. Therefore, the velocity of a galaxy that is 15 Mpc from Earth is $v = 500(15)$ km/s, or 7,500 km/s.

Based on the model, the other choices are incorrect: Choice B is the speed of a galaxy that is 10 Mpc from Earth. Choice C is the speed of a galaxy that is 2.2 Mpc from Earth. Choice D is the speed of a galaxy that is 1.5 Mpc from Earth.

Question 25

Janice puts a fence around her rectangular garden. The garden has a length that is 9 feet less than 3 times its width. What is the perimeter of Janice's fence if the area of her garden is 5,670 square feet?

- A) 342 feet
- B) 318 feet
- C) 300 feet
- D) 270 feet

Item Difficulty: Hard

Content: Passport to Advanced Math

Correct Answer: A

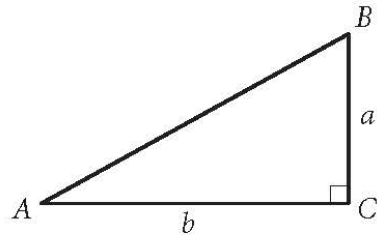
Choice A is the correct answer. Let w represent the width of Janice's garden and $3w - 9$ represent the length of Janice's garden. Since the area of Janice's garden is 5,670 square feet, it follows that $w(3w - 9) = 5,670$, which after dividing by 3 on both sides simplifies to $w(w - 3) = 1,890$.

From this point on, different ways could be used to solve this equation. One could rewrite this quadratic equation in the standard form and use the quadratic formula to solve it. Another approach would be to look among integer factors of 1,890 and try to find two that differ from each other by 3 and whose product is 1,890. The prime factorization of 1,890 ($2 \cdot 3^3 \cdot 5 \cdot 7$) can help with this. Two factors that satisfy the conditions above are 42 and 45 (note that $42 = 2 \cdot 3 \cdot 7$ and $45 = 3^2 \cdot 5$). The numbers -45 and -42 also satisfy the above conditions ($w = -42$), but since w represents the width of Janice's garden, the negative values of w can be rejected. Thus $w = 45$ feet, and so the length of the garden must be $3(45) - 9 = 126$ feet. Therefore, the perimeter of Janice's garden is $2(45 + 126) = 2(171) = 342$ feet.

Choice B is incorrect. This answer choice could result from incorrectly identifying the width of the garden as 42 feet instead of 45 feet. Choices C and D are incorrect; both answers would result in an area of the garden that is significantly smaller than 5,670 square feet. For example, if the perimeter of the garden were 270 feet, as in choice D, then $w + l = 135$ feet, where w represents the width and l represents the length of the garden. So $l = 135 - w$. It is also given that $l = 3w - 9$, which

implies that $135 - w = 3w - 9$. Solving this for w gives $w = 36$, and so $l = 99$. The area of the garden would then be 36×99 square feet, which is clearly less than 5,670 square feet.

Question 26



Given the right triangle ABC above, which of the following is equal to $\frac{b}{a}$?

- A) $\sin A$
- B) $\sin B$
- C) $\tan A$
- D) $\tan B$

Item Difficulty: Hard

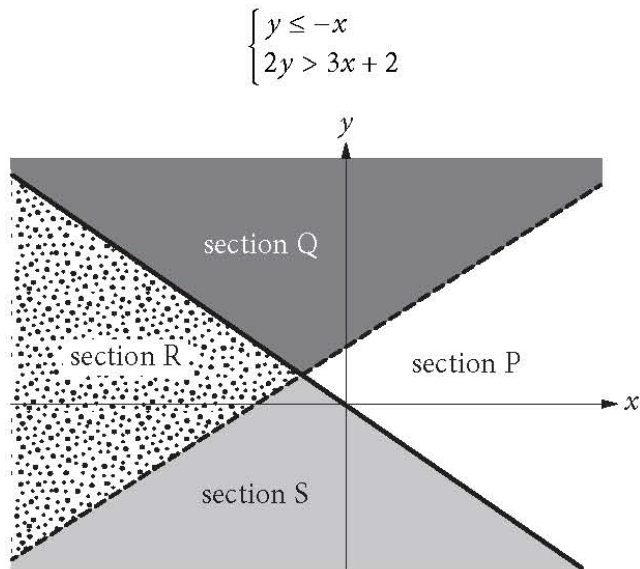
Content: Additional Topics in Math

Correct Answer: D

Choice D is the correct answer. Since the ratio $\frac{b}{a}$ involves only the legs of the right triangle, it follows that, of the given choices, the ratio can be equal to the tangent of one of the angles. In a right triangle, the tangent of an acute angle is defined as the ratio of the opposite side to the adjacent side of the angle. Side b is opposite to angle B and side a is adjacent to angle B . Therefore, $\tan B = \frac{b}{a}$.

Choices A and B cannot be correct; the sine of an acute angle in a right triangle is defined as the ratio of the opposite side to the hypotenuse, and the ratio shown involves only the legs of the triangle. Choice C is incorrect. In the triangle ABC shown, $\tan A = \frac{a}{b}$, not $\frac{b}{a}$.

Question 27



A system of inequalities and a graph are shown above. Which section or sections of the graph could represent all of the solutions to the system?

- A) Section R
- B) Sections Q and S
- C) Sections Q and P
- D) Sections Q, R, and S

Item Difficulty: Hard

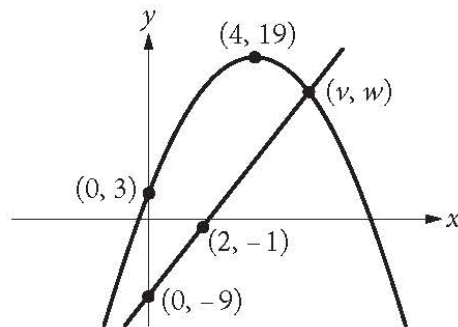
Content: Heart of Algebra

Correct Answer: A

Choice A is the correct answer. The solution set of the inequality $y \leq -x$ is the union of sections R and S of the graph. The solution set of the inequality $2y > 3x + 2$ is the union of sections R and Q of the graph. The solutions of the system consist of the coordinates of all the points that satisfy both inequalities, and therefore, section R represents all the solutions to the system since it is common to the solutions of both inequalities.

Choices B, C, and D are incorrect because they contain ordered pairs that do not satisfy both of the inequalities.

Question 28



The xy -plane above shows one of the two points of intersection of the graphs of a linear function and a quadratic function. The shown point of intersection has coordinates (v, w) . If the vertex of the graph of the quadratic function is at $(4, 19)$, what is the value of v ?

Item Difficulty: Medium

Content: Passport to Advanced Math

The correct answer is 6.

Since the vertex of the graph of the quadratic function is at $(4, 19)$, the equation of the parabola is of the form $y = a(x - 4)^2 + 19$. It is also given that the parabola passes through point $(0, 3)$. This means that

$3 = a(0 - 4)^2 + 19$, and so $a = -1$. So the graph of the parabola is $y = -(x - 4)^2 + 19$.

Since the line passes through the points $(0, -9)$ and $(2, -1)$, one can calculate the slope of the line $\left(\frac{-1 - (-9)}{2 - 0} = 4\right)$ that passes through these points and write the equation of the line in the slope-intercept form as $y = 4x - 9$.

The coordinates of the intersection points of the line and the parabola satisfy both the equation of the parabola and the equation of the line. Therefore, these coordinates are the solutions to the system of equations below:

$$\begin{aligned} y &= 4x - 9 \\ y &= -(x - 4)^2 + 19 \end{aligned}$$

Substituting $4x - 9$ for y into the second equation gives $4x - 9 = -(x - 4)^2 + 19$, which is equivalent to $x^2 - 4x - 12 = 0$. After factoring, this equation can be rewritten as

$(x - 6)(x + 2) = 0$, and so $x = 6$ or $x = -2$. Since point (v, w) is on the right side of the y -axis, it follows that v cannot be -2 . Therefore, $v = 6$.

Question 29

In a college archaeology class, 78 students are going to a dig site to find and study artifacts. The dig site has been divided into 24 sections, and each section will be studied by a group of either 2 or 4 students. How many of the sections will be studied by a group of 2 students?

Item Difficulty: Hard

Content: Heart of Algebra

The correct answer is 9.

Let x be the number of sections that will be studied by 2 students and y be the number of sections that will be studied by 4 students. Since there are 24 sections that will be studied by 78 students, it follows that $x + y = 24$ and $2x + 4y = 78$. Solving this system gives $x = 9$ and $y = 15$. Therefore, 9 of the sections will be studied by a group of 2 students.

Alternatively, if all 24 sections were studied by a group of 4 students, then the total number of students required would be $24 \times 4 = 96$. Since the actual number of students is 78, the difference $96 - 78 = 18$ represents the number of “missing” students, and each pair of these “missing” students represents one of the sections that will be studied by 2 students. Hence, the number of sections that will be studied by 2 students is equal to the number of pairs that 18 students can form, which is $\frac{18}{2} = 9$.

Questions 30 and 31 refer to the following information.

$$v = v_0 - gt \quad (\text{speed-time})$$

$$h = v_0t - \frac{1}{2}gt^2 \quad (\text{position-time})$$

$$v^2 = v_0^2 - 2gh \quad (\text{position-speed})$$

An arrow is launched upward with an initial speed of 100 meters per second (m/s). The equations above describe the constant-acceleration motion of the arrow, where v_0 is the initial speed of the arrow, v is the speed of the arrow as it is moving up in the air, h is the height of the arrow above the ground, t is the time elapsed since the arrow was projected upward, and g is the acceleration due to gravity (9.8 m/s^2).

Question 30

What is the maximum height from the ground the arrow will rise to the nearest meter?

Item Difficulty: Hard

Content: Passport to Advanced Math

The correct answer is 510.

As the arrow moves upward, its speed decreases continuously and it becomes 0 when the arrow reaches its maximum height. Using the position-speed equation and the fact that $v = 0$ when h is maximum gives $0 = 100^2 - 2gh$. Solving for h

gives $h = \frac{100^2}{2(9.8)}$ meters, which to the nearest meter is 510.

Alternatively, the maximum height can be found using the position-time equation.

Substituting 100 for v_0 and 9.8 for g into this equation gives $h = 100t - \frac{1}{2}(9.8)t^2$.

Completing the square gives the equivalent equation

$h = -4.9\left(t - \frac{100}{9.8}\right)^2 + 4.9\left(\frac{100}{9.8}\right)^2$. Therefore, the maximum height from the ground

the arrow will rise is $4.9\left(\frac{100}{9.8}\right)^2$ meters, which to the nearest meter is 510.

Question 31

How long will it take for the arrow to reach its maximum height to the nearest tenth of a second?

Item Difficulty: Hard

Content: Passport to Advanced Math

The correct answer is 10.2 seconds (or $51/5$ seconds).

As the arrow moves upward, its speed decreases continuously, and it becomes 0 when the arrow reaches its maximum height. Using the speed-time equation and the fact that $v = 0$ when h is maximum, we get $0 = 100 - 9.8t$.

Solving this equation for t gives $t = \frac{100}{9.8} = 10.2041$ seconds, which to the nearest tenth of a second is 10.2.