5.3 Solving Trig Equations Practice Worksheet #1
Pre-calculus

Name: _____
Date: _____ Block: ____

Solve for the unknown variable on the interval $0 \le x < 2\pi$.

1. $4\cos^2 x - 3 = 0$

$\cos^2 x = \frac{3}{4}$

$\cos x = \frac{\sqrt{3}}{2}$ $\quad \cos x = \frac{-\sqrt{3}}{2}$

$x = \frac{\pi}{6} + 2\pi n$ $\quad x = \frac{5\pi}{6} + 2\pi n$

$x = \frac{11\pi}{6} + 2\pi n$ $\quad x = \frac{7\pi}{6} + 2\pi n$

2. $\sqrt{2}\sin 2x = 1$

$\sin 2x = \frac{1}{\sqrt{2}}$

$2x = \frac{\pi}{4} + 2\pi n$

$x = \frac{\pi}{8} + \pi n$

$2x = \frac{3\pi}{4} + 2\pi n$

$= \frac{3\pi}{8} + \pi n$

$\frac{\pi}{8}, \frac{9\pi}{8}, \frac{3\pi}{8}, \frac{11\pi}{8}$

3. $3\cot^2 x - 1 = 0$

$\cot^2 x = \frac{1}{3}$

$\tan^2 x = 3$

$\tan x = \sqrt{3}$ $\quad \tan x = -\sqrt{3}$

$x = \frac{\pi}{3} + 2\pi n$ $\quad x = \frac{2\pi}{3} + 2\pi n$

$x = \frac{4\pi}{3} + 2\pi n$ $\quad x = \frac{5\pi}{3} + 2\pi n$

4. $\cos^3 x = \cos x$

$\cos^3 x - \cos x = 0$

$\cos x(\cos^2 x - 1) = 0$

$\cos x = 0$ $\quad \cos^2 x - 1 = 0$

$\cos x = 1$ $\quad \cos x = -1$

$x = \frac{\pi}{2} + 2\pi n$ $\quad x = 0 + 2\pi n$ $\quad x = \pi + 2\pi n$

$x = \frac{3\pi}{2} + 2\pi n$

$0, \pi, \frac{\pi}{2}, \frac{3\pi}{2}$

5. $\sin x - 2\sin x \cos x = 0$

$\sin x(1 - 2\cos x) = 0$

$\sin x = 0$ $\quad 1 - 2\cos x = 0$

$x = 0 + 2\pi n$ $\quad \cos x = \frac{1}{2}$

$x = \pi + 2\pi n$ $\quad x = \frac{\pi}{3} + 2\pi n$

$x = \frac{5\pi}{3} + 2\pi n$

$0, \pi, \frac{\pi}{3}, \frac{5\pi}{3}$

6. $2\sin^2 x - \sin x - 3 = 0$

$(2\sin x - 3)(\sin x + 1) = 0$

$2\sin x - 3 = 0$ $\quad \sin x = -1$

$\sin x = \frac{3}{2}$ $\quad x = \frac{3\pi}{2} + 2\pi n$

No solution

7. $\csc^2 x - \csc x - 2 = 0$

$(\csc x - 2)(\csc x + 1) = 0$

$\csc x = 2$ $\quad \csc x = -1$

$\sin x = \frac{1}{2}$ $\quad \sin x = -1$

$x = \frac{\pi}{6} + 2\pi n$ $\quad x = \frac{3\pi}{2} + 2\pi n$

$x = \frac{5\pi}{6} + 2\pi n$

8. $\cos^2 x = 1 - \sin x$

$1 - \sin^2 x = 1 - \sin x$

$0 = \sin^2 x - \sin x$

$0 = \sin x(\sin x - 1)$

$\sin x = 0$ $\quad \sin x = 1$

$x = 0 + 2\pi n$ $\quad x = \frac{\pi}{2} + 2\pi n$

$x = \pi + 2\pi n$

$0, \pi, \frac{\pi}{2}$

Solve for the unknown variable on the given interval.

9. $\sqrt{3} + \tan(2x) = 0$ on $[0, 2\pi)$.

$\tan(2x) = -\sqrt{3}$

$2x = \frac{2\pi}{3} + 2\pi n$ $\quad 2x = \frac{5\pi}{3} + 2\pi n$

$x = \frac{2\pi}{6} + \pi n$ $\quad x = \frac{5\pi}{6} + \pi n$

$\frac{2\pi}{6}, \frac{8\pi}{6}, \frac{5\pi}{6}, \frac{11\pi}{6}$

10. $\cos(\pi x) = 0.5$ on $[0, 2)$.

$\frac{\pi x}{\pi} = \frac{\frac{\pi}{3} + 2\pi n}{\pi}$

$x = \frac{1}{3} + 2n$

$\pi x = \frac{5\pi}{3} + 2\pi n$

$x = \frac{5}{3} + 2n$

$\frac{1}{3}, \frac{5}{3}$

11. $\sin\left(\frac{x}{2}\right) - 1 = 0$ on $[0, 8\pi)$.

$\sin \frac{x}{2} = 1$

$\frac{x}{2} = \frac{\pi}{2} + 2\pi n$

$x = \pi + 4\pi n$

$\pi, 5\pi$

5.3 Solving Trig Equations – Worksheet #2
Pre-calculus

Name: _____
Date: _____ Block: ___

Part 1: Solve for the unknown variable. Give all of the exact general solutions.

1. $\sin\theta = \frac{\sqrt{2}}{2}$

$\theta = \frac{\pi}{4} + 2\pi n$

$\theta = \frac{3\pi}{4} + 2\pi n$

2. $\frac{\cos\theta}{\cos\theta} = \frac{\sin\theta}{\cos\theta}$

$1 = \tan\theta$

$\theta = \frac{\pi}{4} + 2\pi n$

$\theta = \frac{5\pi}{4} + 2\pi n$

3. $\tan\theta = 1$

same as #2

4. $1 + \sin\theta = 2\cos^2\theta$

$1 + \sin\theta = 2(1 - \sin^2\theta)$

$1 + \sin\theta = 2 - 2\sin^2\theta$

$2\sin^2\theta + \sin\theta - 1 = 0$

$(2\sin\theta - 1)(\sin\theta + 1) = 0$

$\sin\theta = \frac{1}{2}$ $\sin\theta = -1$

$\theta = \frac{\pi}{6} + 2\pi n$ $\theta = \frac{3\pi}{2} + 2\pi n$

$\theta = \frac{5\pi}{6} + 2\pi n$

5. $2\cos^2\theta + \cos\theta = 0$

$\cos\theta(2\cos\theta + 1) = 0$

$\cos\theta = 0$ $2\cos\theta + 1 = 0$

$\cos\theta = -\frac{1}{2}$

$\theta = \frac{\pi}{2} + 2\pi n$

$\theta = \frac{3\pi}{2} + 2\pi n$

$\theta = \frac{2\pi}{3} + 2\pi n$

$\theta = \frac{4\pi}{3} + 2\pi n$

6. $\sin 3\theta = -1$

$3\theta = \frac{3\pi}{2} + 2\pi n$

$\theta = \frac{\pi}{2} + \frac{2\pi n}{3}$

7. $\sin^2\theta - 1 = 0$

$(\sin\theta + 1)(\sin\theta - 1) = 0$

$\sin\theta = -1$ $\sin\theta = 1$

$\theta = \frac{3\pi}{2} + 2\pi n$ $\theta = \frac{\pi}{2} + 2\pi n$

8. $\cos 2\theta = \frac{1}{2}$

$2\theta = \frac{\pi}{3} + 2\pi n$

$\theta = \frac{\pi}{6} + \pi n$

$2\theta = \frac{5\pi}{3} + 2\pi n$

$\theta = \frac{5\pi}{6} + \pi n$

9. $2\sin^2\theta - \sin\theta - 1 = 0$

$(2\sin\theta + 1)(\sin\theta - 1) = 0$

$\sin\theta = -\frac{1}{2}$ $\sin\theta = 1$

$\theta = \frac{7\pi}{6} + 2\pi n$

$\theta = \frac{11\pi}{6} + 2\pi n$

$\theta = \frac{\pi}{2} + 2\pi n$

10. $\tan 4\theta = -1$

$4\theta = \frac{3\pi}{4} + 2\pi n$

$\theta = \frac{3\pi}{16} + \frac{\pi}{2}n$

$4\theta = \frac{7\pi}{4} + 2\pi n$

$\theta = \frac{7\pi}{16} + \frac{\pi}{2}n$

11. $\tan^2 3x = 3$

$\tan 3x = \sqrt{3}$ $\tan 3x = -\sqrt{3}$

$3x = \frac{\pi}{3} + 2\pi n$

$x = \frac{\pi}{9} + \frac{2\pi}{3}n$

$3x = \frac{2\pi}{3} + 2\pi n$

$x = \frac{2\pi}{9} + \frac{2\pi}{3}n$

$3x = \frac{4\pi}{3} + 2\pi n$

$x = \frac{4\pi}{9} + \frac{2\pi}{3}n$

$3x = \frac{5\pi}{3} + 2\pi n$

$x = \frac{5\pi}{9} + \frac{2\pi}{3}n$

12. $\cos\frac{x}{2} = \frac{\sqrt{2}}{2}$

$\frac{x}{2} = \frac{\pi}{4} + 2\pi n$

$x = \frac{\pi}{2} + 4\pi n$

$\frac{x}{2} = \frac{7\pi}{4} + 2\pi n$

$x = \frac{7\pi}{2} + 4\pi n$

Part 2: Solve by approximating the solutions on the interval [0,2π).

13. $2\sin^2 x + 3\sin x + 1 = 0$

$(2\sin x + 1)(\sin x + 1) = 0$

$\sin x = -\frac{1}{2}$ $\quad \sin x = -1$

$x = \frac{7\pi}{6} + 2\pi n$ $\quad x = \frac{3\pi}{2} + 2\pi n$

$x = \frac{11\pi}{6} + 2\pi n$

$\frac{7\pi}{6}, \frac{11\pi}{6}, \frac{3\pi}{2}$

14. $4\sin^2 x = 2\cos x + 1$

$4(1-\cos^2 x) = 2\cos x + 1$

$4 - 4\cos^2 x = 2\cos x + 1$

$0 = 4\cos^2 x + 2\cos x - 3$

$0 =$ [illegible]

15. $\csc x + \cot x = 1$

$\cot x = 1 - \csc x$

$\cot^2 x = \csc^2 x - 2\csc x + 1$

$\csc^2 x - 1 = \csc^2 x - 2\csc x + 1$

$0 = -2\csc x + 2$

$\csc x = 1$

$\sin x = 1$

$x = \frac{\pi}{2} + 2\pi n$

16. $\dfrac{\cos x \cot x}{1 - \sin x} = 3$

17. $\sec^2 x + 0.5\tan x = 1$

Part 3: Use the calculator's inverse trig functions to approximate the solutions. Remember that you must also find the other solution by either adding π, subtracting the value from π, or subtracting the value from 2π.

18. $\tan\theta = 4$

19. $\cos\theta = 0.84$

20. $\sin\theta = 0.63$