

# Practice Worksheet

## Graphs of the Trigonometric Functions

Find each value by referring to the graphs of the trigonometric functions.

1.  $\sin(-720^\circ)$

2.  $\tan(-180^\circ)$

3.  $\cos(540^\circ)$

4.  $\tan(180^\circ)$

5.  $\csc(720^\circ)$

6.  $\sec(180^\circ)$

Find the values of  $\theta$  for which each equation is true.

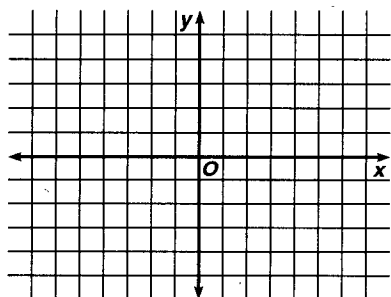
7.  $\sin \theta = -1$

8.  $\sec \theta = -1$

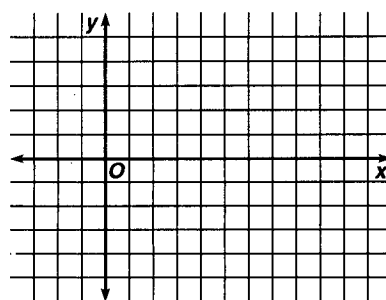
9.  $\tan \theta = 0$

Graph each function on the given interval.

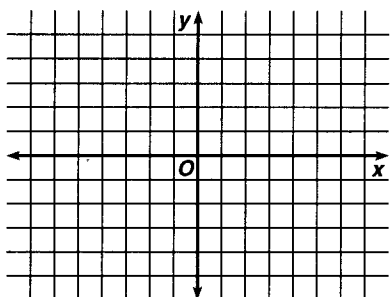
10.  $y = \sin x; -90^\circ \leq x \leq 90^\circ$



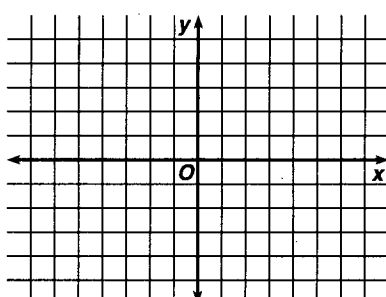
11.  $y = \tan x; -90^\circ \leq x \leq 270^\circ$



12.  $y = \cos x; -360^\circ \leq x \leq 360^\circ$



13.  $y = \sec x; -360^\circ \leq x \leq 360^\circ$



# Practice Worksheet

## Graphs of the Trigonometric Functions

Find each value by referring to the graphs of the trigonometric functions.

1.  $\sin(-720^\circ)$   
0

2.  $\tan(-180^\circ)$   
0

3.  $\cos(540^\circ)$   
-1

4.  $\tan(180^\circ)$   
0

5.  $\csc(720^\circ)$   
undefined

6.  $\sec(180^\circ)$   
-1

Find the values of  $\theta$  for which each equation is true.

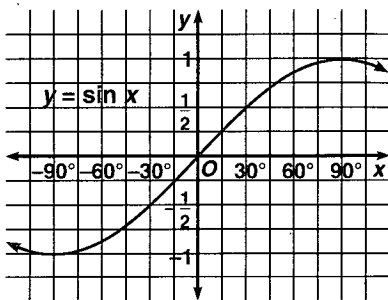
7.  $\sin \theta = -1$   
 $270^\circ + 360k^\circ$   
where  $k$  is any integer

8.  $\sec \theta = -1$   
 $180^\circ + 360k^\circ$   
where  $k$  is any integer

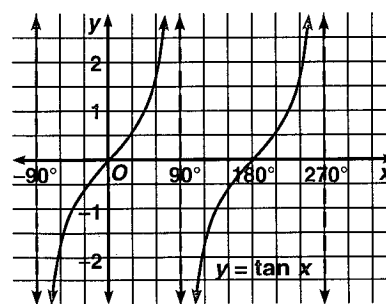
9.  $\tan \theta = 0$   
 $180k^\circ$   
where  $k$  is any integer

Graph each function on the given interval.

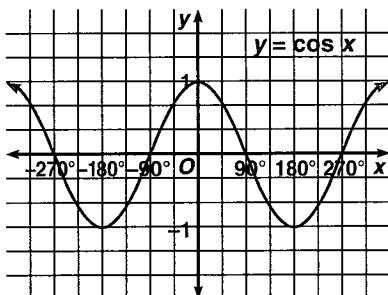
10.  $y = \sin x; -90^\circ \leq x \leq 90^\circ$



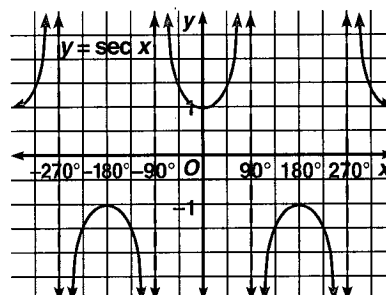
11.  $y = \tan x; -90^\circ \leq x \leq 270^\circ$



12.  $y = \cos x; -360^\circ \leq x \leq 360^\circ$



13.  $y = \sec x; -360^\circ \leq x \leq 360^\circ$



# Practice Worksheet

## Amplitude, Period, and Phase Shift

State the amplitude, period, and phase shift for each function.

1.  $y = -2 \sin \theta$

2.  $y = 10 \sec \theta$

3.  $y = -3 \sin 4\theta$

4.  $y = 0.5 \sin \left( \theta - \frac{\pi}{3} \right)$

5.  $y = 2.5 \cos (\theta + 180^\circ)$

6.  $y = -1.5 \sin \left( 4\theta - \frac{\pi}{4} \right)$

Write an equation of the sine function with each amplitude, period, and phase shift.

7. amplitude = 0.75, period =  $360^\circ$ , phase shift =  $30^\circ$

8. amplitude = 4, period =  $3^\circ$ , phase shift =  $-30^\circ$

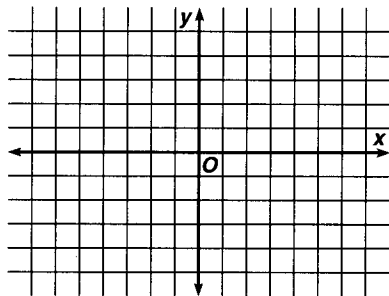
Write an equation of the cosine function with each amplitude, period, and phase shift.

9. amplitude = 3.75, period =  $90^\circ$ , phase shift =  $4^\circ$

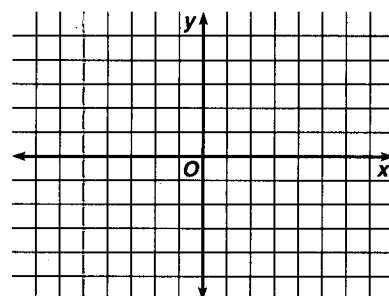
10. amplitude = 12, period =  $45^\circ$ , phase shift =  $180^\circ$

Graph each function.

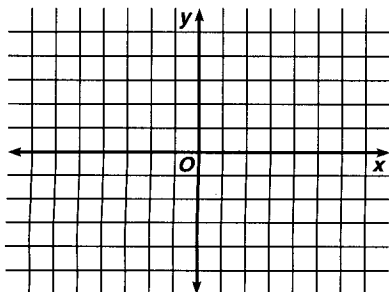
11.  $y = 0.5 \sin x$



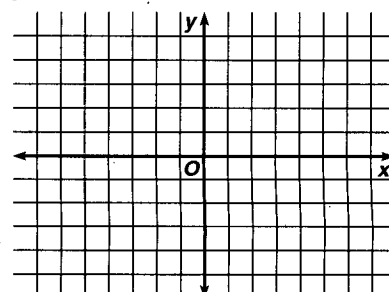
12.  $y = 2 \cos (3x)$



13.  $y = 2 \cos (2x - 45^\circ)$



14.  $y = \tan (x + 60^\circ)$



# Practice Worksheet

## Amplitude, Period, and Phase Shift

State the amplitude, period, and phase shift for each function.

1.  $y = -2 \sin \theta$   
**2,  $360^\circ$ ,  $0^\circ$**

2.  $y = 10 \sec \theta$   
**none,  $360^\circ$ ,  $0^\circ$**

3.  $y = -3 \sin 4\theta$   
**3,  $90^\circ$ ,  $0^\circ$**

4.  $y = 0.5 \sin \left( \theta - \frac{\pi}{3} \right)$   
**0.5,  $360^\circ$ ,  $60^\circ$**

5.  $y = 2.5 \cos (\theta + 180^\circ)$   
**2.5,  $360^\circ$ ,  $-180^\circ$**

6.  $y = -1.5 \sin \left( 4\theta - \frac{\pi}{4} \right)$   
**1.5,  $90^\circ$ ,  $11.25^\circ$**

Write an equation of the sine function with each amplitude, period, and phase shift.

7. amplitude = 0.75, period =  $360^\circ$ , phase shift =  $30^\circ$   
 **$y = 0.75 \sin (\theta - 30^\circ)$  or  $y = -0.75 \sin (\theta - 30^\circ)$**

8. amplitude = 4, period =  $3^\circ$ , phase shift =  $-30^\circ$   
 **$y = \pm 4 \sin (120\theta - 3600^\circ)$**

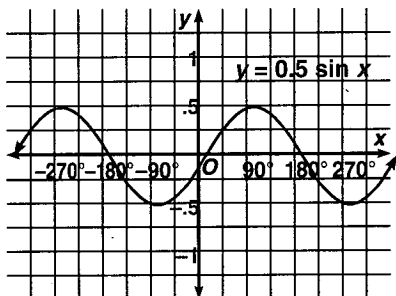
Write an equation of the cosine function with each amplitude, period, and phase shift.

9. amplitude = 3.75, period =  $90^\circ$ , phase shift =  $4^\circ$   
 **$y = \pm 3.75 \cos (4\theta - 16^\circ)$**

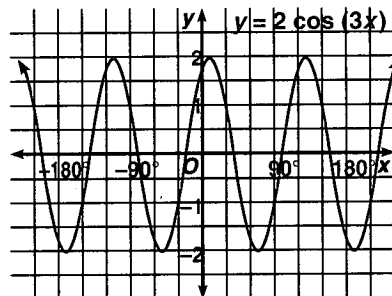
10. amplitude = 12, period =  $45^\circ$ , phase shift =  $180^\circ$   
 **$y = \pm 12 \cos (8\theta - 1440^\circ)$**

Graph each function.

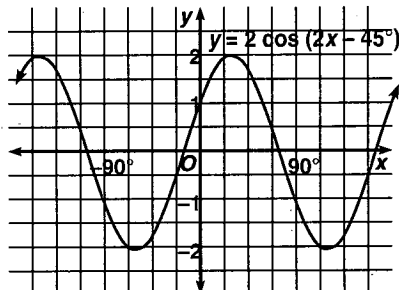
11.  $y = 0.5 \sin x$



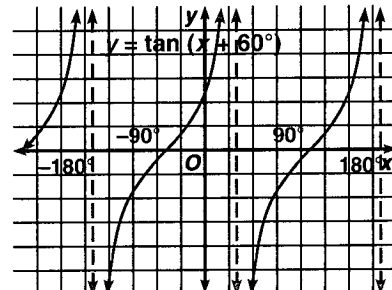
12.  $y = 2 \cos (3x)$



13.  $y = 2 \cos (2x - 45^\circ)$



14.  $y = \tan (x + 60^\circ)$

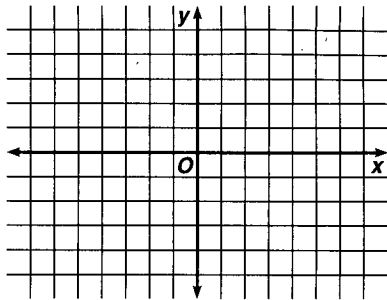


# Practice Worksheet

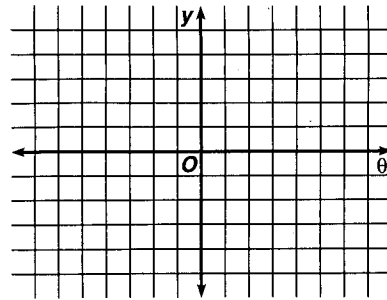
## Graphing Trigonometric Functions

Graph each function.

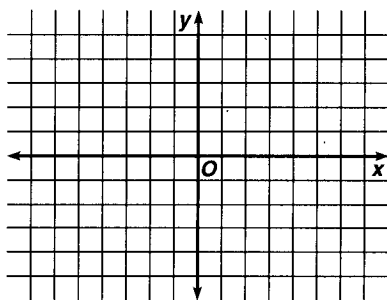
1.  $y = 2 \sin(x - 45^\circ)$



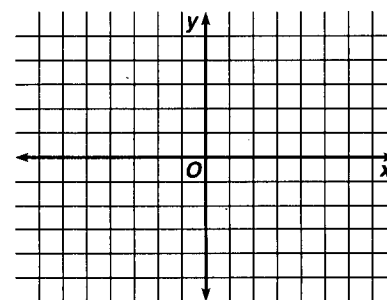
2.  $y = -2 \cos(3\theta)$



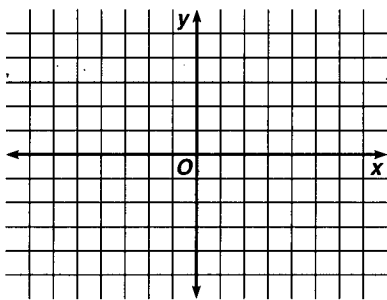
3.  $y = \frac{1}{2} \cos\left(x - \frac{\pi}{2}\right)$



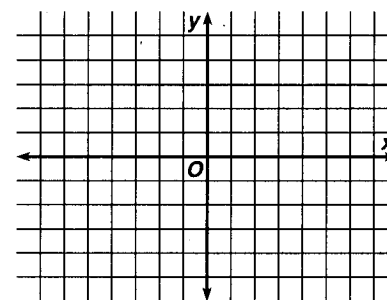
4.  $y = \sin\left(\frac{x}{2} + 90^\circ\right)$



5.  $y = \sin x + \cos x$



6.  $y = \cos 2x - \cos x$

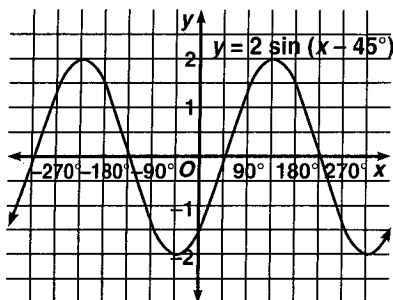


# Practice Worksheet

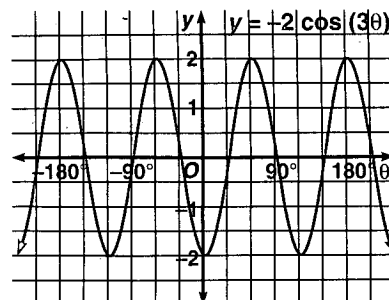
## Graphing Trigonometric Functions

Graph each function.

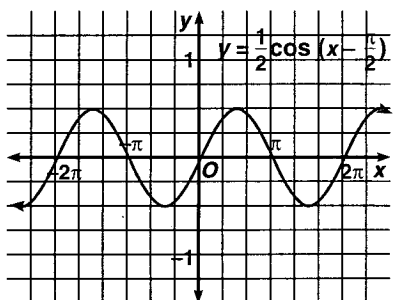
1.  $y = 2 \sin(x - 45^\circ)$



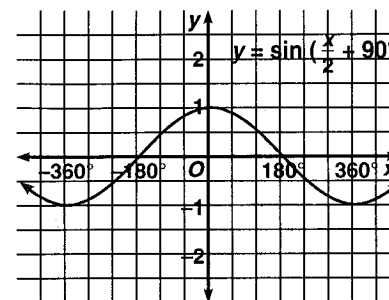
2.  $y = -2 \cos(3\theta)$



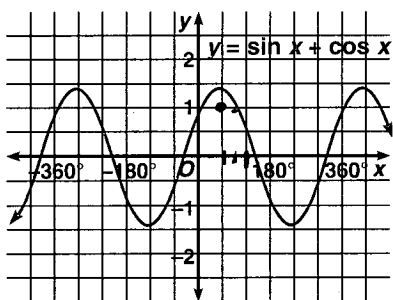
3.  $y = \frac{1}{2} \cos\left(x - \frac{\pi}{2}\right)$



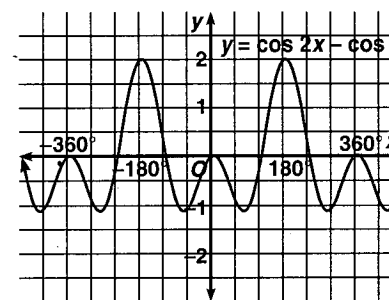
4.  $y = \sin\left(\frac{x}{2} + 90^\circ\right)$



5.  $y = \sin x + \cos x$



6.  $y = \cos 2x - \cos x$



# Practice Worksheet

## Inverse Trigonometric Functions

Write each equation in the form of an inverse relation.

1.  $0.75 = \sin x$

2.  $-1 = \cos x$

3.  $0.1 = \tan \theta$

4.  $\frac{3}{5} = \cos x$

5.  $\sin x = \frac{\sqrt{3}}{2}$

6.  $\cos \alpha = \frac{12}{13}$

Find the values of  $x$  in the interval  $0^\circ \leq x \leq 360^\circ$  that satisfy each equation.

7.  $x = \arccos 1$

8.  $\arccos \frac{\sqrt{2}}{2} = x$

9.  $\arcsin \frac{1}{2} = x$

10.  $\sin^{-1}(-1) = x$

11.  $\sin^{-1} \frac{\sqrt{2}}{2} = x$

12.  $\cot^{-1} 1 = x$

Evaluate each expression. Assume that all angles are in Quadrant I.

13.  $\cos \left( \cos^{-1} \frac{1}{2} \right)$

14.  $\sin \left( \cos^{-1} \frac{1}{2} \right)$

15.  $\cos \left( \sin^{-1} \frac{1}{2} \right)$

16.  $\tan \left( \sin^{-1} \frac{\sqrt{2}}{2} - \cos^{-1} \frac{\sqrt{2}}{2} \right)$

17. Verify that  $\sin^{-1} \frac{\sqrt{3}}{2} + \sin^{-1} \frac{1}{2} = 90^\circ$ . Assume that all angles are in Quadrant I.

# Practice Worksheet

## Inverse Trigonometric Functions

Write each equation in the form of an inverse relation.

1.  $0.75 = \sin x$

$x = \arcsin 0.75$

2.  $-1 = \cos x$

$x = \arccos (-1)$

3.  $0.1 = \tan \theta$

$\theta = \arctan 0.1$

4.  $\frac{3}{5} = \cos x$

$x = \arccos \frac{3}{5}$

5.  $\sin x = \frac{\sqrt{3}}{2}$

$x = \arcsin \frac{\sqrt{3}}{2}$

6.  $\cos \alpha = \frac{12}{13}$

$\alpha = \arccos \frac{12}{13}$

Find the values of  $x$  in the interval  $0^\circ \leq x \leq 360^\circ$  that satisfy each equation.

7.  $x = \arccos 1$

$0^\circ, 360^\circ$

8.  $\arccos \frac{\sqrt{2}}{2} = x$

$45^\circ, 315^\circ$

9.  $\arcsin \frac{1}{2} = x$

$30^\circ, 150^\circ$

10.  $\sin^{-1}(-1) = x$

$270^\circ$

11.  $\sin^{-1} \frac{\sqrt{2}}{2} = x$

$45^\circ, 135^\circ$

12.  $\cot^{-1} 1 = x$

$45^\circ, 225^\circ$

Evaluate each expression. Assume that all angles are in Quadrant I.

13.  $\cos \left( \cos^{-1} \frac{1}{2} \right)$

$0.5$

14.  $\sin \left( \cos^{-1} \frac{1}{2} \right)$

$\frac{\sqrt{3}}{2}$

15.  $\cos \left( \sin^{-1} \frac{1}{2} \right)$

$\frac{\sqrt{3}}{2}$

16.  $\tan \left( \sin^{-1} \frac{\sqrt{2}}{2} - \cos^{-1} \frac{\sqrt{2}}{2} \right)$

$0$

17. Verify that  $\sin^{-1} \frac{\sqrt{3}}{2} + \sin^{-1} \frac{1}{2} = 90^\circ$ . Assume that all angles are in Quadrant I.

$$\sin^{-1} \frac{\sqrt{3}}{2} + \sin^{-1} \frac{1}{2} = 60^\circ + 30^\circ = 90^\circ$$



**Practice Worksheet****Principal Values of the Inverse Trigonometric Functions***Find each value.*

1.  $\text{Arcsin}(-1)$

2.  $\text{Arccos} 1$

3.  $\text{Arctan}(-1)$

4.  $\text{Cos}^{-1} \frac{1}{2}$

5.  $\text{Arcsin} 1$

6.  $\text{Tan}^{-1}\left(-\frac{\sqrt{3}}{3}\right)$

7.  $\cos\left(\text{Cos}^{-1}\left(-\frac{1}{2}\right)\right)$

8.  $\sin\left(\text{Sin}^{-1} \frac{\sqrt{3}}{2}\right)$

9.  $\tan\left(\text{Tan}^{-1} \frac{\sqrt{3}}{3}\right)$

10.  $\text{Cos}^{-1}\left(\text{Cos} \frac{\pi}{2}\right)$

11.  $\text{Sin}^{-1}\left(\sin \frac{\pi}{4}\right)$

12.  $\text{Tan}^{-1}\left(\tan \frac{\pi}{3}\right)$

13.  $\cos\left(\text{Arcsin} \frac{1}{2}\right)$

14.  $\sin\left(\text{Arccos} \frac{3}{2}\right)$

15.  $\tan\left(\text{Arcsin} \frac{\sqrt{3}}{3}\right)$

16.  $\tan\left(\frac{1}{2} \text{Arccos} \frac{5}{13}\right)$

17.  $\cos\left(\frac{1}{2} \text{Arcsin} \frac{6}{10}\right)$

18.  $\sin\left(2 \text{Arccos} \frac{3}{5}\right)$

19.  $\sin\left[\text{Cos}^{-1}\left(\frac{\sqrt{2}}{2}\right) - \frac{\pi}{4}\right]$

20.  $\cos\left[\text{Sin}^{-1}\left(\frac{\sqrt{2}}{2}\right) + \frac{\pi}{4}\right]$

21.  $\text{Tan}\left[\frac{3\pi}{4} + \text{Sin}^{-1} \frac{\sqrt{2}}{2}\right]$

# Practice Worksheet

## Principal Values of the Inverse Trigonometric Functions

Find each value.

1.  $\text{Arcsin}(-1)$   
 $-90^\circ$

2.  $\text{Arccos} 1$   
 $0^\circ$

3.  $\text{Arctan}(-1)$   
 $-45^\circ$

4.  $\text{Cos}^{-1} \frac{1}{2}$   
 $60^\circ$

5.  $\text{Arcsin} 1$   
 $90^\circ$

6.  $\text{Tan}^{-1}\left(-\frac{\sqrt{3}}{3}\right)$   
 $-30^\circ$

7.  $\cos\left(\text{Cos}^{-1}\left(-\frac{1}{2}\right)\right)$   
 $-\frac{1}{2}$

8.  $\sin\left(\text{Sin}^{-1} \frac{\sqrt{3}}{2}\right)$   
 $\frac{\sqrt{3}}{2}$

9.  $\tan\left(\text{Tan}^{-1} \frac{\sqrt{3}}{3}\right)$   
 $\frac{\sqrt{3}}{3}$

10.  $\text{Cos}^{-1}\left(\text{Cos} \frac{\pi}{2}\right)$   
 $\frac{\pi}{2}$

11.  $\text{Sin}^{-1}\left(\sin \frac{\pi}{4}\right)$   
 $\frac{\pi}{4}$

12.  $\text{Tan}^{-1}\left(\tan \frac{\pi}{3}\right)$   
 $\frac{\pi}{3}$

13.  $\cos\left(\text{Arcsin} \frac{1}{2}\right)$   
 $\frac{\sqrt{3}}{2}$

14.  $\sin\left(\text{Arccos} \frac{3}{2}\right)$   
 $\frac{1}{2}$

15.  $\tan\left(\text{Arcsin} \frac{\sqrt{3}}{3}\right)$   
 $\frac{\sqrt{2}}{2}$

16.  $\tan\left(\frac{1}{2} \text{Arccos} \frac{5}{13}\right)$   
 $\frac{2}{3}$

17.  $\cos\left(\frac{1}{2} \text{Arcsin} \frac{6}{10}\right)$   
 $\frac{3\sqrt{10}}{10}$

18.  $\sin\left(2 \text{Arccos} \frac{3}{5}\right)$   
 $\frac{24}{25}$

19.  $\sin\left[\text{Cos}^{-1}\left(\frac{\sqrt{2}}{2}\right) - \frac{\pi}{4}\right]$   
 $0$

20.  $\cos\left[\text{Sin}^{-1}\left(\frac{\sqrt{2}}{2}\right) + \frac{\pi}{4}\right]$   
 $0$

21.  $\text{Tan}\left[\frac{3\pi}{4} + \text{Sin}^{-1} \frac{\sqrt{2}}{2}\right]$   
 $0$

# Practice Worksheet

## Graphing Inverses of Trigonometric Functions

State the domain and range of each relation.

1.  $y = \sin x + 1$

2.  $y = \sin x + 1$

3.  $y = \cos x - 1$

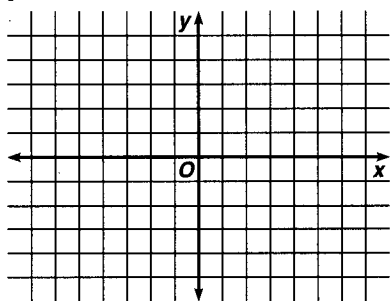
4.  $y = \cos^{-1} x$

5.  $y = \arcsin x$

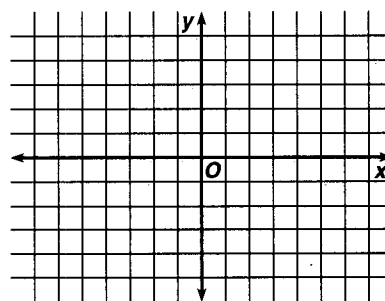
6.  $y = \tan^{-1} x$

Write the equation for the inverse of each function. Then graph the function and its inverse.

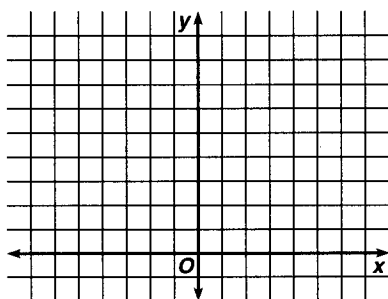
7.  $y = \cos^{-1} x$



8.  $y = \tan^{-1}(3x)$



9.  $y = \frac{\pi}{2} + \cos^{-1} x$



10.  $y = \sin\left(x - \frac{\pi}{2}\right)$



Determine if each of the following is true or false. If false, give a counterexample.

11.  $\cos^{-1} x = \cos^{-1}(-x)$

12.  $\sin^{-1} x = -\sin^{-1} x$

# Practice Worksheet

## Graphing Inverses of Trigonometric Functions

State the domain and range of each relation.

1.  $y = \sin x + 1$

domain:

$-90^\circ \leq x \leq 90^\circ$

range:  $0 \leq y \leq 2$

2.  $y = \sin x + 1$

domain:

all real numbers

range:  $0 \leq y \leq 2$

3.  $y = \cos x - 1$

domain:

all real numbers

range:  $-2 \leq y \leq 0$

4.  $y = \cos^{-1} x$

domain:

$-1 \leq x \leq 1$

range:  $0^\circ \leq y \leq 180^\circ$

5.  $y = \arcsin x$

domain:

$-1 \leq x \leq 1$

range: all real numbers

6.  $y = \tan^{-1} x$

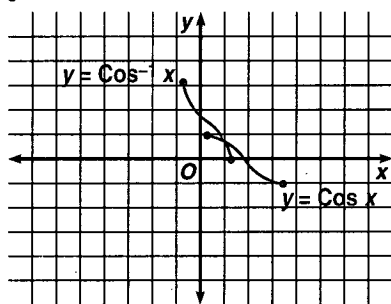
domain:

all real numbers

range:  $-90^\circ \leq y \leq 90^\circ$

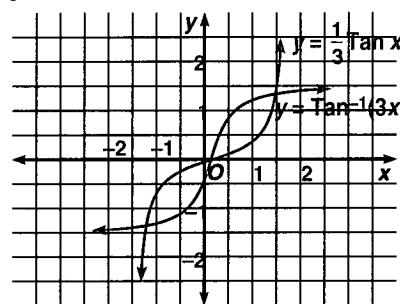
Write the equation for the inverse of each function. Then graph the function and its inverse.

7.  $y = \cos^{-1} x$



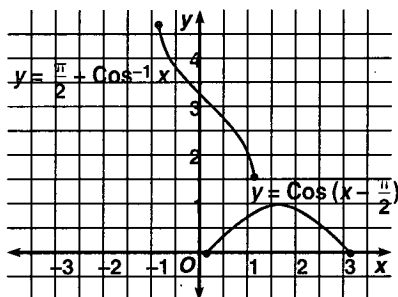
$y = \cos x$

8.  $y = \tan^{-1}(3x)$



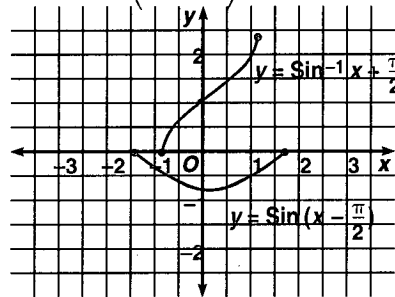
$y = \frac{1}{3} \tan x$

9.  $y = \frac{\pi}{2} + \cos^{-1} x$



$y = \cos\left(x - \frac{\pi}{2}\right)$

10.  $y = \sin\left(x - \frac{\pi}{2}\right)$



$y = \sin^{-1} x + \frac{\pi}{2}$

Determine if each of the following is true or false. If false, give a counterexample.

11.  $\cos^{-1} x = \cos^{-1}(-x)$  false; Let  $x = 1$ .  $\cos^{-1} 1 = 0^\circ$  but  $\cos^{-1}(-1) = 180^\circ$ . So,  $\cos^{-1} x \neq \cos^{-1}(-x)$  for all  $x$ .

12.  $\sin^{-1} x = -\sin^{-1} x$  false; Let  $x = 1$ .  $\sin^{-1} 1 = 90^\circ$  but  $-\sin^{-1} 1 = -90^\circ$ . So,  $\sin^{-1} x \neq -\sin^{-1} x$  for all  $x$ .

# Practice Worksheet

## Simple Harmonic Motion

Find the amplitude, period, frequency, and phase shift.

1.  $y = 3 \sin\left(\frac{1}{2}\theta - 90^\circ\right)$

2.  $y = -\frac{1}{2} \cos \theta$

3.  $y = \sin\left(x + \frac{3\pi}{2}\right)$

4.  $y = -2 \sin\left(2x - \frac{\pi}{6}\right)$

5.  $y = 1.5 \cos\left(\frac{\pi}{6}x + \frac{\pi}{3}\right)$

6.  $y = 12 \cos\left(2\pi t - \frac{\pi}{6}\right)$

7.  $w = -220 \cos 12t$

8.  $z = -0.3 \sin\left(\frac{3\pi t}{4} - \frac{\pi}{4}\right)$

Write an equation with phase shift 0 to represent simple harmonic motion under each set of circumstances.

9. initial position 12, amplitude 12, period 8

10. initial position 0, amplitude 2, period  $8\pi$

11. initial position  $-24$ , amplitude 24, period 6

# Practice Worksheet

## Simple Harmonic Motion

Find the amplitude, period, frequency, and phase shift.

$$1. y = 3 \sin\left(\frac{1}{2}\theta - 90^\circ\right)$$

$$3, 720^\circ, \frac{1}{720}, 180^\circ$$

$$3. y = \sin\left(x + \frac{3\pi}{2}\right)$$

$$1, 2\pi, \frac{1}{2\pi}, -\frac{3\pi}{2}$$

$$5. y = 1.5 \cos\left(\frac{\pi}{6}x + \frac{\pi}{3}\right)$$

$$1.5, 12, \frac{1}{12}, -2$$

$$7. w = -220 \cos 12t$$

$$220, 30^\circ, \frac{1}{30}, 0^\circ$$

$$2. y = -\frac{1}{2} \cos \theta$$

$$\frac{1}{2}, 360^\circ, \frac{1}{360}, 0^\circ$$

$$4. y = -2 \sin\left(2x - \frac{\pi}{6}\right)$$

$$2, \pi, \frac{1}{\pi}, \frac{\pi}{12}$$

$$6. y = 12 \cos\left(2\pi t - \frac{\pi}{6}\right)$$

$$12, 1, 1, \frac{1}{12}$$

$$8. z = -0.3 \sin\left(\frac{3\pi t}{4} - \frac{\pi}{4}\right)$$

$$0.3, \frac{8}{3}, \frac{3}{8}, \frac{1}{3}$$

Write an equation with phase shift 0 to represent simple harmonic motion under each set of circumstances.

9. initial position 12, amplitude 12, period 8

$$y = 12 \cos \frac{\pi t}{4}$$

10. initial position 0, amplitude 2, period  $8\pi$

$$y = \pm 2 \sin \frac{t}{4}$$

11. initial position -24, amplitude 24, period 6

$$y = -24 \cos \frac{\pi t}{3}$$

**Practice Worksheet****Basic Trigonometric Identities***Solve for values of  $\theta$  between  $0^\circ$  and  $90^\circ$ .*

1. If  $\tan \theta = 2$ , find  $\cot \theta$ .

2. If  $\sin \theta = \frac{2}{3}$ , find  $\cos \theta$ .

3. If  $\cos \theta = \frac{1}{4}$ , find  $\tan \theta$

4. If  $\tan \theta = 3$ , find  $\sec \theta$ .

5. If  $\sin \theta = \frac{7}{10}$ , find  $\cot \theta$ .

6. If  $\tan \theta = \frac{7}{2}$ , find  $\sin \theta$ .

*Express each value as a function of an angle in Quadrant I.*

7.  $\sin 458^\circ$

8.  $\cos 892^\circ$

9.  $\tan (-876^\circ)$

10.  $\csc 495^\circ$

*Simplify.*

11.  $\frac{\cot A}{\tan A}$

12.  $\frac{\sin^2 \beta \cot \beta}{\cos \beta}$

13.  $\sin^2 \theta \cos^2 \theta - \cos^2 \theta$

14.  $\cos x + \sin x \tan x$

# Practice Worksheet

## Basic Trigonometric Identities

Solve for values of  $\theta$  between  $0^\circ$  and  $90^\circ$ .

1. If  $\tan \theta = 2$ , find  $\cot \theta$ .

$$\frac{1}{2}$$

2. If  $\sin \theta = \frac{2}{3}$ , find  $\cos \theta$ .

$$\frac{\sqrt{5}}{3}$$

3. If  $\cos \theta = \frac{1}{4}$ , find  $\tan \theta$

$$\sqrt{15}$$

4. If  $\tan \theta = 3$ , find  $\sec \theta$ .

$$\sqrt{10}$$

5. If  $\sin \theta = \frac{7}{10}$ , find  $\cot \theta$ .

$$\frac{51}{7}$$

6. If  $\tan \theta = \frac{7}{2}$ , find  $\sin \theta$ .

$$\frac{7\sqrt{53}}{53}$$

Express each value as a function of an angle in Quadrant I.

7.  $\sin 458^\circ$   
 **$\sin 82^\circ$**

8.  $\cos 892^\circ$   
 **$-\cos 8^\circ$**

9.  $\tan(-876^\circ)$   
 **$\tan 24^\circ$**

10.  $\csc 495^\circ$   
 **$\csc 45^\circ$**

Simplify.

11.  $\frac{\cot A}{\tan A}$

$$\cot^2 A$$

12.  $\frac{\sin^2 \beta \cot \beta}{\cos \beta}$

$$\sin \beta$$

13.  $\sin^2 \theta \cos^2 \theta - \cos^2 \theta$   
 **$-\cos^4 \theta$**

14.  $\cos x + \sin x \tan x$   
 **$\sec x$**



# Practice Worksheet

## Verifying Trigonometric Identities

Verify that each of the following is an identity.

1.  $\frac{\csc x}{\cot x + \tan x} = \cos x$

2.  $\sin^3 x - \cos^3 x = (1 + \sin x \cos x)(\sin x - \cos x)$

3.  $\frac{1}{\sin y - 1} - \frac{1}{\sin y + 1} = -2\sec^2 y$

4.  $1 - 2 \sin^2 r + \sin^4 r = \cos^4 r$

5.  $\tan u + \frac{\cos u}{1 + \sin u} = \sec u$

6.  $\frac{\tan x + \sec x}{\sec x - \cos x + \tan x} = \csc x$

Find a numerical value of one trigonometric function of each  $x$ .

7.  $\sin x = 3 \cos x$

8.  $\cos x = \cot x$

# Practice Worksheet

## Verifying Trigonometric Identities

Verify that each of the following is an identity.

$$1. \frac{\csc x}{\cot x + \tan x} = \cos x$$

$$\begin{aligned} \frac{\csc x}{\cot x + \tan x} &= \frac{\frac{1}{\sin x}}{\frac{\cos x}{\sin x} + \frac{\sin x}{\cos x}} \cdot \frac{\sin x \cos x}{\sin x \cos x} = \frac{\cos x}{\cos^2 x \sin^2 x} \\ &= \frac{\cos x}{1} = \cos x \end{aligned}$$

$$\begin{aligned} 2. \sin^3 x - \cos^3 x &= (1 + \sin x \cos x)(\sin x - \cos x) \\ \sin^3 x - \cos^3 x &= (\sin x - \cos x)(\sin^2 x + \sin x \cos x + \cos^2 x) \\ &= (\sin x - \cos x)(1 + \sin x \cos x) \end{aligned}$$

$$3. \frac{1}{\sin y - 1} - \frac{1}{\sin y + 1} = -2\sec^2 y$$

$$\frac{1}{\sin y - 1} - \frac{1}{\sin y + 1} = \frac{\sin y + 1 - \sin y - 1}{\sin^2 y - 1} = \frac{2}{-\cos^2 y} = -2 \sec^2 y$$

$$\begin{aligned} 4. 1 - 2\sin^2 r + \sin^4 r &= \cos^4 r \\ \cos^4 r &= (1 - \sin^2 r)^2 = 1 - 2\sin^2 r + \sin^4 r \end{aligned}$$

$$5. \tan u + \frac{\cos u}{1 + \sin u} = \sec u$$

$$\begin{aligned} \tan u + \frac{\cos u}{1 + \sin u} &= \frac{\sin u}{\cos u} + \frac{\cos u}{1 + \sin u} = \frac{\sin u + \sin^2 u + \cos^2 u}{(\cos u)(1 + \sin u)} \\ &= \frac{1 + \sin u}{(\cos u)(1 + \sin u)} = \sec u \end{aligned}$$

$$6. \frac{\tan x + \sec x}{\sec x - \cos x + \tan x} = \csc x$$

$$\frac{\sin x + 1}{1 - \cos^2 x + \sin x} = \frac{\sin x + 1}{\sin^2 x + \sin x} = \frac{\sin x + 1}{(\sin x)(\sin x + 1)} = \csc x$$

Find a numerical value of one trigonometric function of each  $x$ .

$$\begin{aligned} 7. \sin x &= 3 \cos x \\ \tan x &= 3 \end{aligned}$$

$$\begin{aligned} 8. \cos x &= \cot x \\ \csc x &= 1 \text{ or } \sin x = 1 \end{aligned}$$

**Practice Worksheet****Sum and Difference Identities**

*Use the sum and difference identities to find the exact value of each function.*

1.  $\cos 75^\circ$

2.  $\cos 375^\circ$

3.  $\sin (-165^\circ)$

4.  $\sin (-105^\circ)$

5.  $\sin 95^\circ \cos 55^\circ + \cos 95^\circ \sin 55^\circ$

6.  $\cos 160^\circ \cos 40^\circ + \sin 160^\circ \sin 40^\circ$

7.  $\tan (135^\circ + 120^\circ)$

8.  $\tan 345^\circ$

*If  $\alpha$  and  $\beta$  are the measures of two first quadrant angles, find the exact value of each function.*

9. If  $\sin \alpha = \frac{12}{13}$  and  $\cos \beta = \frac{3}{5}$ , find  $\cos (\alpha - \beta)$ .

10. If  $\cos \alpha = \frac{12}{13}$  and  $\cos \beta = \frac{12}{37}$ , find  $\tan (\alpha - \beta)$ .

11. If  $\cos \alpha = \frac{8}{17}$  and  $\tan \beta = \frac{5}{12}$ , find  $\cos (\alpha + \beta)$ .

12. If  $\csc \alpha = \frac{13}{12}$  and  $\sec \beta = \frac{5}{3}$ , find  $\sin (\alpha - \beta)$ .

*Verify that each of the following is an identity.*

13.  $\cos (180^\circ - \theta) = -\cos \theta$

14.  $\sin (360^\circ + \theta) = \sin \theta$

# Practice Worksheet

## Sum and Difference Identities

Use the sum and difference identities to find the exact value of each function.

1.  $\cos 75^\circ$

$$\frac{\sqrt{6} - \sqrt{2}}{4}$$

2.  $\cos 375^\circ$

$$\frac{\sqrt{6} + \sqrt{2}}{4}$$

3.  $\sin (-165^\circ)$

$$\frac{\sqrt{2} - \sqrt{6}}{4}$$

4.  $\sin (-105^\circ)$

$$\frac{-\sqrt{2} - \sqrt{6}}{4}$$

5.  $\sin 95^\circ \cos 55^\circ + \cos 95^\circ \sin 55^\circ$

$$\frac{1}{2}$$

6.  $\cos 160^\circ \cos 40^\circ + \sin 160^\circ \sin 40^\circ$

$$-\frac{1}{2}$$

7.  $\tan (135^\circ + 120^\circ)$

$$2 + \sqrt{3}$$

8.  $\tan 345^\circ$

$$\sqrt{3} - 2$$

If  $\alpha$  and  $\beta$  are the measures of two first quadrant angles, find the exact value of each function.

9. If  $\sin \alpha = \frac{12}{13}$  and  $\cos \beta = \frac{3}{5}$ , find  $\cos (\alpha - \beta)$ .  $\frac{63}{65}$

10. If  $\cos \alpha = \frac{12}{13}$  and  $\cos \beta = \frac{12}{37}$ , find  $\tan (\alpha - \beta)$ .  $-\frac{360}{319}$

11. If  $\cos \alpha = \frac{8}{17}$  and  $\tan \beta = \frac{5}{12}$ , find  $\cos (\alpha + \beta)$ .  $\frac{21}{221}$

12. If  $\csc \alpha = \frac{13}{12}$  and  $\sec \beta = \frac{5}{3}$ , find  $\sin (\alpha - \beta)$ .  $\frac{16}{65}$

Verify that each of the following is an identity.

13.  $\cos (180^\circ - \theta) = -\cos \theta$

$$\begin{aligned} &\cos (180^\circ - \theta) \\ &= \cos 180^\circ \cos \theta + \sin 180^\circ \sin \theta \\ &= (-1) \cos \theta + 0 \cdot \sin \theta \\ &= -\cos \theta \end{aligned}$$

14.  $\sin (360^\circ + \theta) = \sin \theta$

$$\begin{aligned} &\sin (360^\circ + \theta) \\ &= \sin 360^\circ \cos \theta + \cos 360^\circ \sin \theta \\ &= 0 \cdot \cos \theta + 1 \cdot \sin \theta \\ &= \sin \theta \end{aligned}$$

**Practice Worksheet****Double-Angle and Half-Angle Identities**

If  $\sin A = \frac{12}{13}$  and  $A$  is in the first quadrant, find each value.

1.  $\cos 2A$

2.  $\sin 2A$

3.  $\tan 2A$

4.  $\cos \frac{A}{2}$

5.  $\sin \frac{A}{2}$

6.  $\tan \frac{A}{2}$

Use a half-angle identity to find each value.

7.  $\tan \frac{\pi}{8}$

8.  $\cos \frac{5\pi}{8}$

9.  $\sin \frac{19\pi}{12}$

10.  $\cos 67\frac{1}{2}^\circ$

Verify that each of the following is an identity.

11.  $\tan \frac{A}{2} = \frac{\sin A}{1 + \cos A}$

12.  $\tan \frac{A}{2} = \frac{1 - \cos A}{\sin A}$

# Practice Worksheet

## Double-Angle and Half-Angle Identities

If  $\sin A = \frac{12}{13}$  and  $A$  is in the first quadrant, find each value.

1.  $\cos 2A$

$$-\frac{119}{169}$$

2.  $\sin 2A$

$$\frac{120}{169}$$

3.  $\tan 2A$

$$-\frac{120}{119}$$

4.  $\cos \frac{A}{2}$

$$\frac{3\sqrt{13}}{13}$$

5.  $\sin \frac{A}{2}$

$$\frac{2\sqrt{13}}{13}$$

6.  $\tan \frac{A}{2}$

$$\frac{2}{3}$$

Use a half-angle identity to find each value.

7.  $\tan \frac{\pi}{8}$

$$\sqrt{\frac{2 - \sqrt{2}}{2 + \sqrt{2}}}$$

8.  $\cos \frac{5\pi}{8}$

$$\frac{\sqrt{2}}{2}$$

9.  $\sin \frac{19\pi}{12}$

$$-\frac{\sqrt{2 + \sqrt{3}}}{2}$$

10.  $\cos 67\frac{1}{2}^\circ$

$$\frac{\sqrt{2 - \sqrt{2}}}{2}$$

Verify that each of the following is an identity.

11.  $\tan \frac{A}{2} = \frac{\sin A}{1 + \cos A}$

$$\begin{aligned} \tan \frac{A}{2} &= \sqrt{\frac{1 - \cos A}{1 + \cos A}} \cdot \sqrt{\frac{1 + \cos A}{1 + \cos A}} \\ &= \sqrt{\frac{1 - \cos^2 A}{(1 + \cos A)^2}} \\ &= \frac{\sin A}{1 + \cos A} \end{aligned}$$

12.  $\tan \frac{A}{2} = \frac{1 - \cos A}{\sin A}$

$$\begin{aligned} \tan \frac{A}{2} &= \frac{\sin A}{1 + \cos A} \cdot \frac{1 - \cos A}{1 - \cos A} \\ &= \frac{\sin A (1 - \cos A)}{1 - \cos^2 A} \\ &= \frac{\sin A (1 - \cos A)}{\sin^2 A} = \frac{1 - \cos A}{\sin A} \end{aligned}$$

**Practice Worksheet****Solving Trigonometric Equations**

*Solve each equation for all values of  $x$ .*

1.  $2 \sin^2 x - 5 \sin x + 2 = 0$

2.  $\sin^2 x - 2 \sin x - 3 = 0$

3.  $3 \cos 2x - 5 \cos x = 1$

4.  $2 \tan x \cos x + 2 \cos x = \tan x + 1$

*Solve each equation for  $0^\circ \leq x \leq 180^\circ$ .*

5.  $2 \sin^2 x - 1 = 0$

6.  $\cos x = 3 \cos x - 2$

7.  $\tan x = \sin x$

8.  $\cos x \sin 2x = 0$

9.  $\sec x = 1 + \tan x$

10.  $4 \sin^2 x - 4 \sin x + 1 = 0$

11.  $\sin 2x = 2 \cos x$

12.  $\tan^2 x + \tan x = 0$

13.  $2 \sin 2x = 1$

14.  $\cos 2x + \sin x = 1$

# Practice Worksheet

## Solving Trigonometric Equations

Solve each equation for all values of  $x$ .

1.  $2 \sin^2 x - 5 \sin x + 2 = 0$   
 $30^\circ + 360k^\circ, 150^\circ + 360k^\circ$

2.  $\sin^2 x - 2 \sin x - 3 = 0$   
 $270^\circ + 360k^\circ$

3.  $3 \cos 2x - 5 \cos x = 1$   
 $120^\circ + 360k^\circ, 240^\circ + 360k^\circ$

4.  $2 \tan x \cos x + 2 \cos x = \tan x + 1$   
 $-60 + 360k^\circ, -45^\circ + 180k^\circ$   
 $60^\circ + 360k^\circ$

Solve each equation for  $0^\circ \leq x \leq 180^\circ$ .

5.  $2 \sin^2 x - 1 = 0$   
 $45^\circ, 135^\circ$

6.  $\cos x = 3 \cos x - 2$   
 $0^\circ$

7.  $\tan x = \sin x$   
 $0^\circ, 180^\circ$

8.  $\cos x \sin 2x = 0$   
 $0^\circ, 90^\circ, 180^\circ$

9.  $\sec x = 1 + \tan x$   
 $0^\circ$

10.  $4 \sin^2 x - 4 \sin x + 1 = 0$   
 $30^\circ, 150^\circ$

11.  $\sin 2x = 2 \cos x$   
 $90^\circ$

12.  $\tan^2 x + \tan x = 0$   
 $0^\circ, 135^\circ, 180^\circ$

13.  $2 \sin 2x = 1$   
 $15^\circ, 75^\circ$

14.  $\cos 2x + \sin x = 1$   
 $0^\circ, 30^\circ, 150^\circ, 180^\circ$