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- I. Model Problems.
- II. Practice
- III. Challenge Problems
- IV. Answer Key

### Web Resources

#### *Inverse Functions*

[www.mathwarehouse.com/algebra/relation/inverse-of-function.php](http://www.mathwarehouse.com/algebra/relation/inverse-of-function.php)

#### *Functions and Relations*

[www.mathwarehouse.com/algebra/relation/](http://www.mathwarehouse.com/algebra/relation/)

We Recommend [Meta Calculator --a free online graphing calculator](#)



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## Inverse Functions

The function  $g(x)$  are inverses of each other  $f(x)$  if  $g(f(x)) = x$  and  $f(g(x)) = x$ .

The inverse of the function  $f(x)$  is indicated with the notation  $f^{-1}(x)$ , read  $f$  inverse (this notation does **not** mean  $\frac{1}{f(x)}$ ).

### I. Model Problems

In this example we will find the inverse of a discrete function for a given as a list of ordered pairs.

**Example 1:** If  $f = \{(3, 2), (4, -6), (-2, 11), (5, 5)\}$  find  $f^{-1}(x)$ .

When finding the inverse exchange  $x$  and  $y$ . The ordered pairs  $(x, y)$  become  $(y, x)$ .

**Answer:**  $f^{-1} = \{(2, 3), (-6, 4), (11, -2), (5, 5)\}$

In these examples we will find the inverse of functions given as an equation.

**Example 2:** If  $f(x) = 3x + 10$  find  $f^{-1}(x)$ .

Write function in terms of  $y$ .

$$\begin{aligned} f(x) &= 3x + 10 \\ y &= 3x + 10 \end{aligned}$$

When finding the inverse exchange  $x$  and  $y$ .

Solve for  $y$ .

$$\begin{aligned} x &= 3y + 10 \\ -10 & \quad -10 \end{aligned}$$

$$\frac{x - 10}{3} = \frac{3y}{3}$$

$$\frac{x - 10}{3} = y$$

Rewrite as  $f^{-1}(x)$ .

$$f^{-1}(x) = \frac{x - 10}{3}$$

**Answer:**  $f^{-1}(x) = \frac{x - 10}{3}$

**Example 3:** If  $f(x) = \sqrt{x + 12}$  find  $f^{-1}(x)$ .

Write function in terms of  $y$ .

$$\begin{aligned} f(x) &= \sqrt{x + 12} \\ y &= \sqrt{x + 12} \end{aligned}$$

When finding the inverse exchange  $x$  and  $y$ .

Solve for  $y$ . Square both sides of the equation.

$$x = \sqrt{y + 12}$$

$$x^2 = (\sqrt{y + 12})^2$$

$$x^2 = y + 12$$

$$\begin{aligned} -12 & \quad -12 \end{aligned}$$

$$x^2 - 12 = y$$

$$f^{-1}(x) = x^2 - 12$$

Rewrite as  $f^{-1}(x)$ .

**Answer:**  $f^{-1}(x) = x^2 - 12, x \in \mathbb{R} \mid x \geq 0$

## II. Practice Problems

Solve.

1. Is  $g(x) = \frac{1}{2}x - 2$  the inverse of  $f(x) = 2x + 4$ ? Justify your answer.
2. Is  $g(x) = 4x + 24$  the inverse of  $f(x) = \frac{1}{4}x + 6$ ? Justify your answer.
3. Is  $h(x) = x^2 - 2$  the inverse of  $g(x) = \sqrt{x + 2}$ ? Justify your answer.
4. Is  $h(x) = x^2$  the inverse of  $g(x) = \sqrt{x}$ ? Justify your answer.

Find the inverse of the given function.

5.  $f = \{(1,3), (2,-5), (3,6)\}$
6.  $g = \{(-4,1), (-3,2), (0,0), (1,10)\}$
7.  $h = \{(-1,-1), (0,0), (3,3), (6,6)\}$
- 8.

$x$	$y$
-3	-2
-1	2
0	4
1	6
3	8

- 9.
10.  $f(x) = 3x - 7$

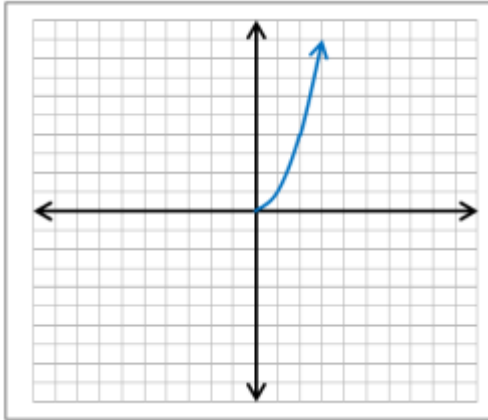
$x$	$y$
-3	0
1	2
6	3
13	4
22	5

11.  $g(x) = -4x + 5$
12.  $h(x) = \frac{2}{5}x + 6$
13.  $f(x) = \frac{3x+4}{7}$
14.  $g(x) = \frac{1}{4}x + 6$
15.  $g(x) = -3x - 10$
16.  $f(x) = \sqrt{x - 4}$
17.  $g(x) = \sqrt{2x + 8}$
18.  $h(x) = \sqrt{3x} - 6$
19.  $f(x) = 4\sqrt{x}$
20. Graph the inverse of  $f(x) = 4x - 12$ .

### Challenge Problems

1. Graph the inverse of  $f(x) = \sqrt{x+1}$  (Hint: identify the domain of  $f(x)$ ).

2. Graph the inverse of the function graphed below.



scale of x and y-axes is 1.

3. Find the inverse of the function  $f(x) = \sqrt[3]{x+4}$ .

4. Find the error in the student's work for the following problem:

If  $f(x) = \frac{x-7}{x}$ , find  $f^{-1}(x)$ .

Given  $f(x) = \frac{x-7}{x}$

Step 1  $y = \frac{x}{x-7}$

Step 2  $x = \frac{x}{y-7}$

Step 3  $x(x) = \left(\frac{y-7}{x}\right)x$

Step 4  $x^2 = \frac{y-7}{+7}$

Step 5  $x^2 + 7 = y$

Step 6  $f^{-1}(x) = x^2 + 7$

5. Find the inverse of the function  $f(x) = \frac{x-2}{x}$ .

#### IV. Answer Key

1. yes;  $g(f(x)) = g(2x + 4) = \frac{1}{2}(2x + 4) - 2 = x + 2 - 2 = x$ ;

$$f(g(x)) = g\left(\frac{1}{2}x - 2\right) = 2\left(\frac{1}{2}x - 2\right) + 4 = x - 4 + 4 = x$$

2. no;  $g(f(x)) = g\left(\frac{1}{4}x + 6\right) = 4\left(\frac{1}{4}x + 6\right) + 24 = x + 24 + 24 = x + 48 \neq x$

3. no;  $h(g(x)) = h(\sqrt{x+2}) = (\sqrt{x+2})^2 - 2 = x + 2 - 2 = x$ ;

$$g(h(x)) = g(x^2 - 2) = \sqrt{(x^2 - 2) + 2} = \sqrt{x^2} = x \quad (x > 0)$$

while  $h(g(x))=x$ ,  $g(h(x))=abs(x)$

4. no;  $h(g(x)) = h(\sqrt{x}) = (\sqrt{x})^2 = x$ ;  $g(h(x)) = g(x^2) = \sqrt{x^2} = x \quad (x > 0)$

while  $h(g(x))=x$ ,  $g(h(x))=abs(x)$

5.  $f^{-1} = \{(3,1), (-5,2), (6,3)\}$

6.  $g^{-1} = \{(1,-4), (2,-3), (0,0), (10,1)\}$

7.  $h^{-1} = \{(-1,-1), (0,0), (3,3), (6,6)\}$

8.

x	y
-2	-3
2	-1
4	0
6	1
8	3

9.

x	y
0	-3
2	1
3	6
4	13
5	22

10.  $f^{-1}(x) = \frac{x+7}{3}$

11.  $g^{-1}(x) = \frac{x-5}{4}$

12.  $h^{-1}(x) = \frac{5}{2}x - 15$

13.  $f^{-1}(x) = \frac{7x-4}{3}$

14.  $g^{-1}(x) = 4x + 24$

15.  $g^{-1}(x) = \frac{x+10}{3}$

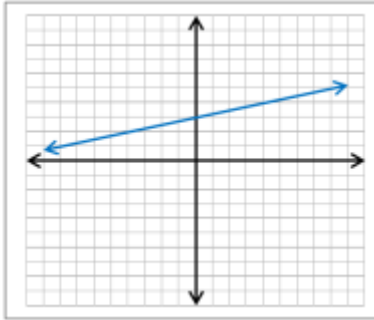
16.  $f^{-1}(x) = x^2 - 4$ ,  $x \in \mathbb{R} \mid x \geq 0$

17.  $g^{-1}(x) = \frac{x^2-8}{2}$ ,  $x \in \mathbb{R} \mid x \geq 0$

18.  $h^{-1}(x) = \frac{x^2+12x+36}{3}$ ,  $x \in \mathbb{R} \mid x \geq -6$

19.  $f(x) = \frac{x^2}{16}$ ,  $x \in \mathbb{R} \mid x \geq 0$

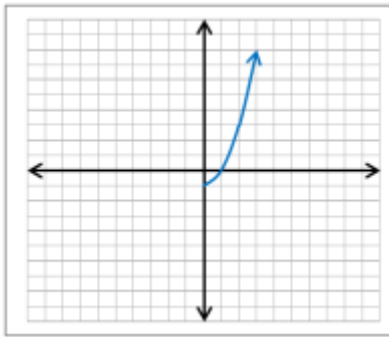
20.



scale of x and y-axes is 1.

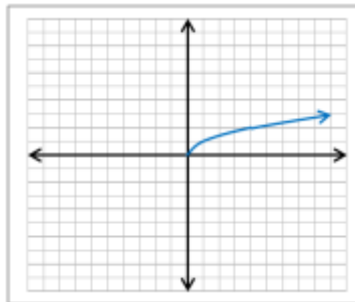
### Challenge Problems

1.



scale of x and y-axes is 1.

2.



scale of x and y-axes is 1.

3.  $g^{-1}(x) = x^3 - 4$

4. Step 2; replace both  $x$  with  $y$

5.  $f^{-1}(x) = \frac{-2}{x-1}$