MATH 083 Final Exam Review

Completing the problems in this review will greatly prepare you for the final exam. Calculator use is not required, but you are permitted to use a calculator during the final exam period. Calculators with advanced capabilities (such as the TI-89) or cell phones are not permitted. The final exam is cumulative and contains 25 questions worth four points each.

1) Factor each polynomial completely.
   a) $48x^5y^7+12x^4y^6+16x^4y$
   b) $27y^2-75$
   c) $6y^2+y-15$
   d) $2y^3-6y^2+5y-15$
   e) $-16m^2n^3+24mn^2-8mn$
   f) $x^2+13x-48$
   g) $4y^2+20y+25$
   h) $27t^3-125$
   i) $2m^3+14m^2+24m$
   j) $3x^2+13x+4$

2) Find all numbers for which each rational expression is undefined, if any.
   a) $\dfrac{12}{7x-21}$
   b) $\dfrac{x+9}{5}$
   c) $\dfrac{3}{4x}$
   d) $\dfrac{x+1}{x^2+5x+6}$

3) Simplify each rational expression, if possible.
   a) $\dfrac{48m^7}{20m^4}$
   b) $\dfrac{3x+12}{x^2-16}$
   c) $\dfrac{y^2+3y}{2y^2+7y+3}$
   d) $\dfrac{3-m}{2m^2-18}$

4) Perform the indicated operation and simplify, if possible.
   a) $\dfrac{9x+17}{2x+5}-\dfrac{3x+2}{2x+5}$
   b) $\dfrac{a}{a^2-5a+4}+\dfrac{2}{a^2-5a+4}+\dfrac{a^2-4a}{a^2-5a+4}$
   c) $\dfrac{-2x}{x^2+7x+12}+\dfrac{5}{4x+16}$
   d) $\dfrac{4x}{x^2+2x+1}-\dfrac{2x+5}{x^2+4x+3}$
   e) $\dfrac{6x+30}{10x^2-40}\cdot\dfrac{x^2-3x-10}{x^2-25}$
   f) $\dfrac{y^2+16y+64}{3y^2+25y+8}\div\dfrac{-3y-24}{9y^2-1}$

5) Solve.
   a) $\dfrac{t+1}{2t-1}=\dfrac{5}{7}$
   b) $\dfrac{2}{p+1}-\dfrac{1}{p-1}=\dfrac{2p}{p^2-1}$
   c) $\dfrac{1}{m-3}+2=\dfrac{1}{m-3}$
   d) $\dfrac{n+1}{n^2+2n-3}=\dfrac{n}{n+3}-\dfrac{1}{n-1}$

6) Determine whether each relation is a function. Identify the domain and range for each relation.
   a) $\{(2,7),(4,-9),(5,10),(9,7),(0,-6)\}$
   b) $\{(-6,-7),(0,9),(1,3),(-6,1)\}$

7) Consider the functions $f(x)=2x^2+11x+15$; $g(x)=2x+5$; and $h(x)=\dfrac{12}{x-3}$. Determine:

a) $f(-3)$.

b) $(f+g)(x)$.

c) $(f-g)(x)$.

d) $(f\cdot g)(x)$.

e) $\left(\dfrac{f}{g}\right)(x)$.

f) $(f+h)(-3)$.

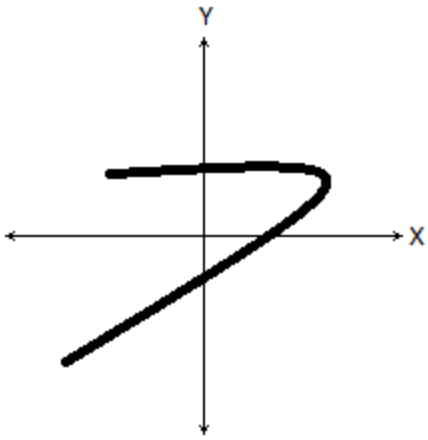
g) $(g-f)(2)$.

h) $(g\cdot h)(1)$.

i) $\left(\dfrac{f}{g}\right)(0)$.

8) Does each graph represent *y* as a function of *x*?

a)



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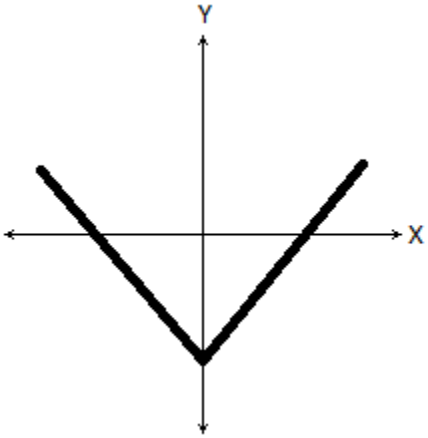

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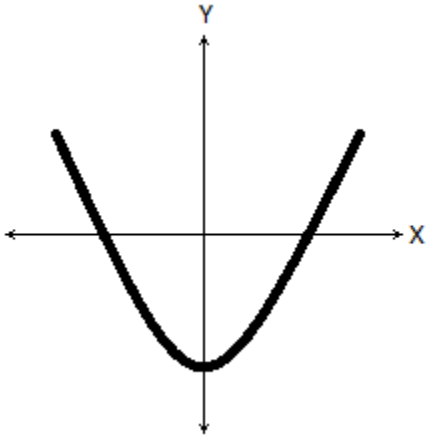

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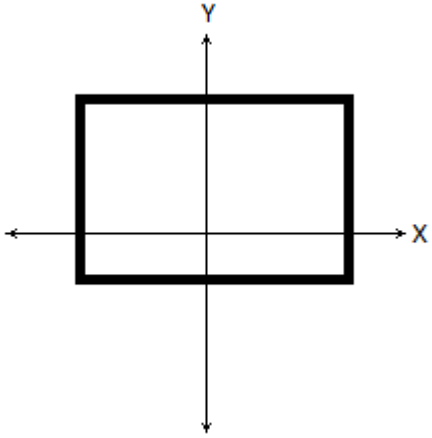


9) State the domain of each function.

a) $f(x)=\dfrac{2x-1}{x^2+10x+24}$

b) $g(x)=\sqrt{5x-3}$

c) $h(x)=3x^2+5x-4$

d) $g(x)=3^x$

e) $f(x)=\log_2 x$

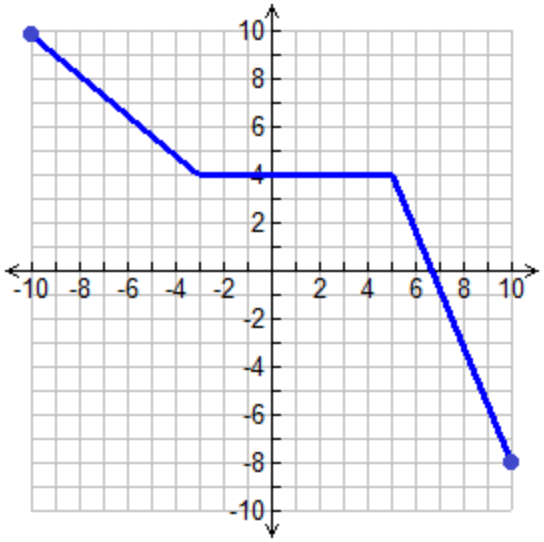
10) Use the given graph of $f(x)$ on the following page to answer the exercises below.

a) Determine $f(-9)$.

b) Determine $f(5)$.

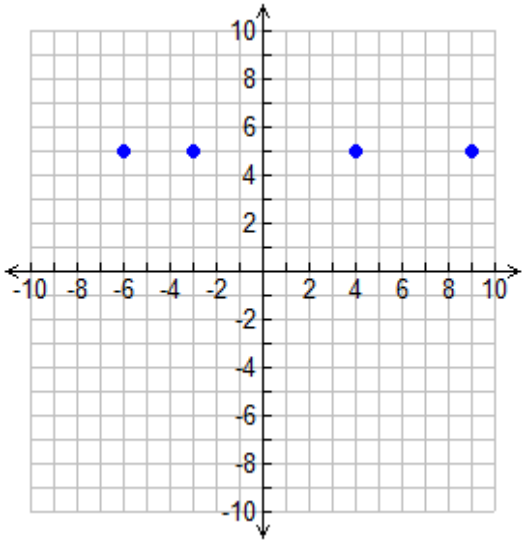
c) For what value of *x* is $f(x)=-6$?

d) For what value of *x* is $f(x)=9$?



11) Use the graph of each function to identify its domain and range.

a)



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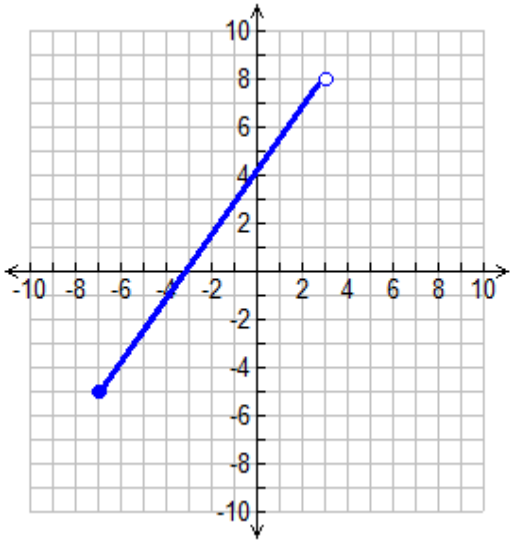

12) Use radical notation to rewrite each expression. Simplify, if possible.

a) $16^{\frac{3}{2}}$

b) $(4ab)^{\frac{1}{3}}$

c) $32^{-\frac{3}{5}}$

d) $(7x^5)^{\frac{1}{2}}$

13) Simplify. Assume that all variables in a radicand represent positive real numbers.

a) $\sqrt{64m^{16}n^{22}}$

b) $\sqrt{48x^{17}y^8z^3}$

c) $\sqrt[3]{108a^{14}b^{16}c}$

d) $\sqrt[3]{24m^9n^{14}}$

14) Write the expression using a single radical.

a) $\sqrt{x}\cdot\sqrt[3]{x}$

b) $\dfrac{\sqrt[5]{n^4}}{\sqrt[3]{n^2}}$

15) Add or subtract, as indicated. Assume that all variables in a radicand represent positive real numbers.

a) $2\sqrt[3]{5}-7\sqrt[3]{5}+\sqrt[3]{5}$

b) $3\sqrt{20x^2}+\sqrt{112y^6}-2\sqrt{80x^2}+2\sqrt{28y^6}$

c) $\sqrt{80y}-\sqrt{125y}$

d) $2\sqrt[3]{16}+7\sqrt[3]{250}-3\sqrt[3]{54}$

16) Multiply and simplify, if possible. Assume that all variables in a radicand represent positive real numbers.

a) $\sqrt{5}\left(3-\sqrt{7}\right)$

b) $\sqrt{7}\left(\sqrt{2}+3\sqrt{14}\right)$

c) $6\sqrt{2y}\left(3\sqrt{2y}+2\sqrt{10y}\right)$

d) $\left(7+\sqrt{3}\right)\left(6-2\sqrt{5}\right)$

e) $\left(2\sqrt{a}+\sqrt{b}\right)\left(\sqrt{a}+\sqrt{b}\right)$

f) $\left(1-\sqrt{2}\right)^2$

g) $\left(x+\sqrt{3}\right)^2$

17) Simplify, if possible. Assume that all variables in a radicand represent positive real numbers

a) $\dfrac{\sqrt{150}}{\sqrt{6}}$

b) $\dfrac{\sqrt{45a^7b^{13}}}{\sqrt{5a^2b}}$

c) $\sqrt[4]{\dfrac{9m^3}{16n^{12}}}$

18) Rationalize each denominator.

a) $\dfrac{9}{\sqrt{5x}}$

b) $\sqrt{\dfrac{4}{10}}$

c) $\dfrac{3}{\sqrt{2}}$

d) $\dfrac{1+\sqrt{7}}{\sqrt{2}}$

19) Solve.

a) $\sqrt{4x+1}=5$

b) $7+\sqrt[3]{3y+10}=5$

c) $\sqrt{2x+5}-1=x$

d) $\sqrt{x+25}=\sqrt{5x-3}$

20) Express each number in terms of i and simplify, if possible.

a) $\sqrt{-36}$

b) $-\sqrt{-49}$

c) $\sqrt{-80}$

d) $2\sqrt{-45}+3\sqrt{-20}$

21) Perform the indicated operation. Write the result in the form $a+bi$.

a) $(2+7i)+(1-3i)$

b) $(10-8i)-(6-9i)$

c) $7i(1+2i)$

d) $(2-5i)(4+i)$

e) $\dfrac{7}{6i}$

f) $\dfrac{6}{2+i}$

g) $\dfrac{1+2i}{3-4i}$

22) Solve each equation by factoring.

a) $x^2 + x - 56 = 0$

b) $9m^2 = 25$

c) $3n^2 + 17n = -10$

d) $y^2 + 8 = 3y(y-2)$

23) Solve each equation by using the square root property.

a) $5r^2 - 3 = 77$

b) $y^2 + 24 = 0$

c) $(3x-5)^2 - 6 = 30$

d) $(2n+5)^2 = 75$

24) Solve each equation by completing the square.

a) $x^2 + 8x + 12 = 0$

b) $n^2 - 3n = 4$

c) $3x^2 - 12x - 84 = 0$

d) $x^2 - 9x + 4 = x - 25$

25) Solve each equation by using the quadratic formula.

a) $x^2 - 6x - 1 = 0$

b) $2y^2 = -5y + 12$

c) $4x^2 - 10x = -7$

d) $(x+2)(x+1) = 4$

26) Solve each equation by the method of your choice.

a) $5n^2 - 11 = 29$

b) $4x^2 + 3x = 7 + 6x$

c) $5(3n+2)^2 - 90 = 0$

d) $3p^2 - 6p = -24$

27) Find the vertex and all *x*- and *y*-intercepts of the given functions. Use this information to sketch the graph of each function. Then, identify the domain and range of each function.

a) $f(x) = x^2 - 2x - 3$

b) $g(x) = -x^2 - 4x + 5$

28) Sketch the graph of a quadratic function with vertex $(-3,-1)$ and intercepts $(-4,0), (-2,0)$, and $(0,8)$.

29) Set up a table of coordinates for the function. Use the coordinates to sketch the graph of each function. Then, identify the domain and range of each function.

a) $f(x) = 2^x$

b) $g(x) = \left(\frac{1}{3}\right)^x$

c) $h(x) = 2^x - 3$

d) $f(x) = 4^{-x}$

e) $g(x) = \log_2 x$

30) Write each equation in its equivalent exponential form.

a) $\log_2 16 = 4$

b) $-5 = \log_3\left(\frac{1}{243}\right)$

c) $\log 1000 = 3$

d) $\ln e = 1$

31) Write each equation in its equivalent logarithmic form.

a) $6^{-3} = \frac{1}{216}$

b) $7^x = 30$

c) $\left(\frac{1}{4}\right)^2 = \frac{1}{16}$

d) $45 = a^2$

32) Evaluate each expression.

a) $\log_7 343$

b) $\log_8\left(\frac{1}{8}\right)$

33) Solve.

a) $2^x = 16$

b) $36^x = 216$

c) $9^{x+2} = 81$

d) $8^{2x-1} = 32^{x-3}$

e) $5^{2x+1} = 125^{2x}$

f) $\log_3 x = 4$

g) $\log_m\left(\frac{1}{16}\right) = -4$

h) $\log_7 1 = n$

34) A boat travels 60 miles per hour in still water. Find the speed of the river's current if the boat traveled 80 miles down the river in the same time that it took to travel 70 miles up the river.

35) Paul is preparing to complete quarterly tax reports for his employer. Paul can complete the reports in 12 hours if he works alone. His coworker Sharese can complete the reports in 6 hours if she works alone. How long will it take to complete the reports if Paul and Sharese work together?

36) A tire company's revenue, *R*, in dollars is directly proportional to the number of tires, *n*, it sells. In a particular month, the company generated revenue of $136,710 on the sale of 3,255 tires. What is the company's monthly revenue if 4,000 tires are sold?

37) The length of a sound wave, *w*, in meters is inversely proportional to the frequency, *f*, of the sound in hertz (Hz). Bottlenose dolphins emit clicking sounds at different frequencies for communicating, orienting themselves to their surroundings, avoiding obstacles, and finding food. If the wavelength of a click that has a frequency of 300 Hz is 5.1 meters, find the wavelength of a click that has a frequency of 500 Hz.

38) A stone is thrown upward with an initial velocity of 48 feet per second from a bridge 280 feet above a river. The height of the stone above the river *t* seconds after it is thrown is given by the function $s(t) = -16t^2 + 48t + 280$. When does the stone reach its maximum height? What is the maximum height?

39) The population of rabbits in a barn, *P*, after *t* weeks is modeled the function $P(t) = 5 \cdot 2^t$. How many rabbit are in the barn after 6 weeks?