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MATH 117 Right Triangle Word Problems

Here we shall consider various word problems that provide some physical applications of the right-triangle trig formulas.

Right-Triangle Formulas

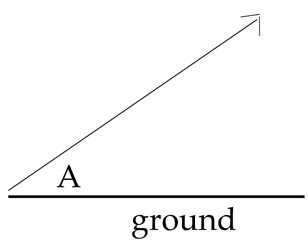
$$x^2 + y^2 = z^2 \qquad z = \sqrt{x^2 + y^2} \qquad x = \sqrt{z^2 - y^2} \qquad y = \sqrt{z^2 - x^2}$$

$$\cos\theta = \frac{\text{Adj}}{\text{Hyp}} = \frac{x}{z} \qquad \sin\theta = \frac{\text{Opp}}{\text{Hyp}} = \frac{y}{z} \qquad \tan\theta = \frac{\text{Opp}}{\text{Adj}} = \frac{y}{x}$$

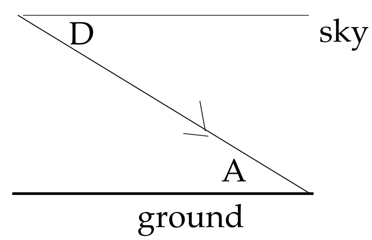
$$x = z\cos\theta \text{ and } y = z\sin\theta$$

Angles of Elevation and Depression

We often measure an angle from the ground upward (an angle of elevation), or downward *from* an imaginary horizontal line (an angle of depression). But by alternate interior angles, an angle of depression will be congruent to an angle of elevation.



An Angle of Elevation A



An Angle of Depression D
Then D ≅ A

Instructions: With each problem, draw a diagram and completely label all necessary pieces. Establish a right-triangle trigonometric relationship. Give the *exact* algebraic solution in terms of the given information, then give a numerical approximation.

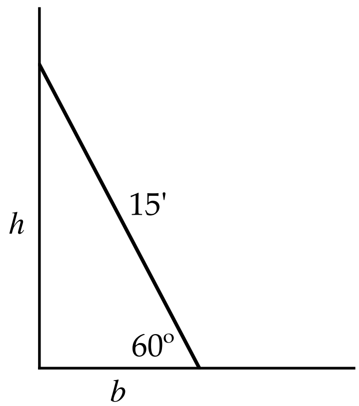
Example 1. (i) A 15 ft ladder leans against a wall at an angle of elevation of 60º.

(a) How high up the wall does the ladder rest?
(b) How far from the wall is the base of the ladder?

(ii) A 50 ft pole has a support wire that runs from its top to the ground with an angle of depression of 75º.

(a) How far from the base of the pole does the wire connect to the ground?
(b) How much wire is used?

Solutions. (i)

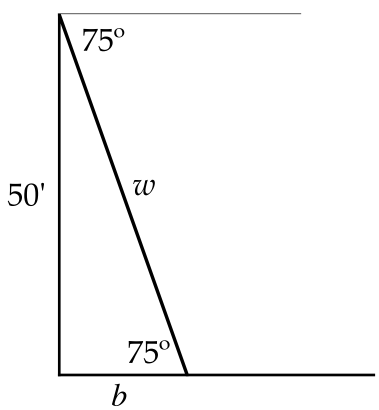


Let the height be h and the base be b. Then,

(a) $\sin 60° = \dfrac{h}{15} \;\rightarrow\; h = 15 \sin 60° \approx \mathbf{12.99\ ft}$

(b) $\cos 60° = \dfrac{b}{15} \;\rightarrow\; b = 15 \cos 60° = \mathbf{7.5\ ft}$

(ii)



Let the base be b and the wire be w. Then,

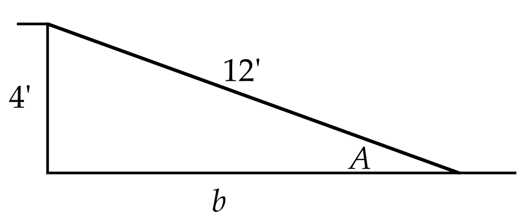
(a) $\tan 75° = \dfrac{50}{b} \;\rightarrow\; b = \dfrac{50}{\tan 75°} \approx \mathbf{13.4\ ft}$

(b) $\sin 75° = \dfrac{50}{w} \;\rightarrow\; w = \dfrac{50}{\sin 75°} \approx \mathbf{51.76\ ft}$

Example 2. A flat 12 foot plank rests with one end on the ground and the other end upon a 4 foot ledge.

(a) How far from the base of the ledge is the far end of the plank?
(b) What is the grade (i.e., angle of elevation)?

Solution.

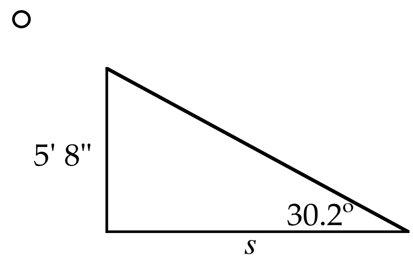


(a) $4^2 + b^2 = 12^2 \;\rightarrow\; b = \sqrt{12^2 - 4^2} \approx \mathbf{11.3\ ft}$

(b) $\sin A = \dfrac{4}{12} \;\rightarrow\; A = \sin^{-1}\left(\dfrac{4}{12}\right) \approx \mathbf{19.47°}$

Example 3. Jamie is 5' 8" tall. Find the length of her shadow if the angle of elevation of the sun is 30.2°.

Solution.



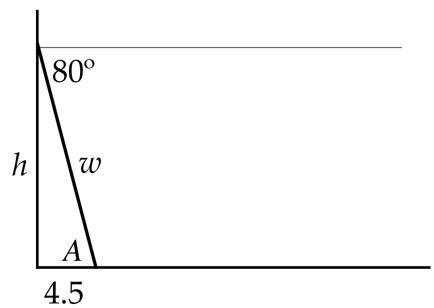
Let s be the length of her shadow. Then

$$\tan(30.2^\circ) = \frac{(5 + 8/12)}{s}$$

$$\to s = \frac{(5+8/12)}{\tan(30.2^\circ)} \approx \mathbf{9.7363\ ft.}$$

Example 4. Some wire connects from a pole to point on the ground at an angle of depression of 80°. On the ground, the wire is 4.5 ft from the pole. (a) How much wire is used? (b) How high up the pole is the wire connected?

Solution.



$A = 80^\circ$ also

Let the height be h and the wire be w.

$$\text{(a)}\ \cos(80^\circ) = \frac{4.5}{w}$$

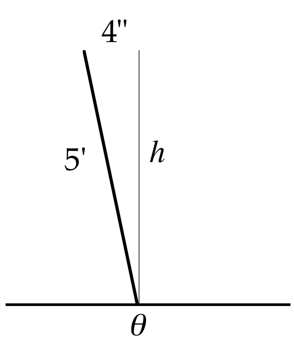
$$\to w = \frac{4.5}{\cos(80^\circ)} \approx \mathbf{25.9\ ft.}$$

$$\text{(b)}\ \tan(80^\circ) = \frac{h}{4.5}$$

$$\to h = 4.5\tan(80^\circ) \approx \mathbf{25.52\ ft.}$$

Example 5. A 5 ft post is supposed to be vertical, but it is 4 inches out of alignment at the top. (a) What is the angle of lean? (b) How high does it reach?

Solution.



$$\text{(a)}\ \sin\theta = \frac{(4/12)}{5} = \frac{1}{15}$$

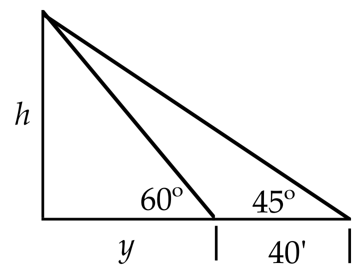
$$\to \theta = \sin^{-1}\left(\frac{1}{15}\right) \approx \mathbf{3.822^\circ}.$$

$$\text{(b)}\ h^2 + (4/12)^2 = 5^2$$

$$\to h = \sqrt{5^2 - (1/3)^2} \approx \mathbf{4.99\ ft}$$

Example 6. At a certain distance, the angle of elevation to the top of a building is 60°. From 40 feet further back, the angle of elevation is 45°. Find the height of the building.

Solution.



$$\tan(60^\circ) = \frac{h}{y} \quad \text{and} \quad \tan(45^\circ) = \frac{h}{40 + y}$$

$$\rightarrow h = y\tan(60^\circ) = (40 + y)\tan(45^\circ)$$

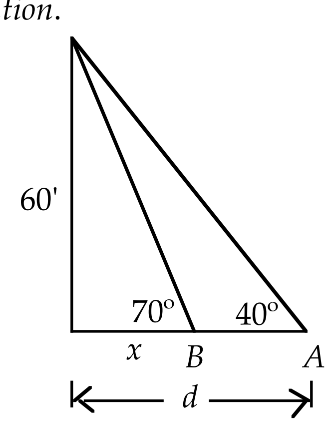
Thus, $y\tan(60^\circ) - y\tan(45^\circ) = 40\tan(45^\circ)$

$$\rightarrow y = \frac{40\tan(45^\circ)}{\tan(60^\circ) - \tan(45^\circ)}$$

Then, $h = y\tan(60^\circ) = \dfrac{40\tan(45^\circ) \times \tan(60^\circ)}{\tan(60^\circ) - \tan(45^\circ)} \approx$ **94.64 ft**.

Example 7. A building is 60 ft high. From a distance at point A on the ground, the angle of elevation to the top of the building is 40°. From a little nearer at point B, the angle of elevation is 70°. Find the distance from point A to point B.

Solution.



$$\tan(70^\circ) = \frac{60}{x} \quad \text{and} \quad \tan(40^\circ) = \frac{60}{d}$$

$$\rightarrow x = \frac{60}{\tan(70^\circ)} \quad \text{and} \quad d = \frac{60}{\tan(40^\circ)}$$

The distance from point A to point B is

$d - x = \dfrac{60}{\tan(40^\circ)} - \dfrac{60}{\tan(70^\circ)} \approx$ **49.667 ft**.