Sampling Distributions Worksheet – Review Chapter 18

1. Taxi Fares are normally distributed with mean fare \$22.27 and a standard deviation of \$2.20.
A) Which should have the greater probability of falling between \$21 & \$24 – the mean of a random sample of 10 taxi fares or the amount of a single random taxi fare? Why?

B) Which should have a greater probability of being over \$24 – the mean of 10 randomly selected taxi fares or the amount of a single randomly selected taxi fare? Why?

2. Suppose a sample of 50 MP3 players is drawn randomly from a population of MP3 players and the weight, x, of each MP3 player is recorded. Prior experience has shown that the weight of a single MP3 player has a mean of 6 ounces and a standard deviation of 2.5 ounces.
A) Describe the shape of the sampling distribution of $\bar{x}$ and justify your answer.

B) What is the mean and standard deviation of the sampling distribution?

C) What is the probability that the sample has a mean weight of less than 5 ounces?

D) How would the sampling distribution of $\bar{x}$ change if the sample size, n , were increased from 50 to 100?

3. A soft-drink bottle vendor claims that its process yields bottles with a mean internal strength of 157 psi (pounds per square inch) and a standard deviation of 3 psi and is normally distributed. As part of its vendor surveillance, a bottler strikes an agreement with the vendor that permits the bottler to sample from the vendor's production to verify the vendor's claim.
A) Suppose the bottler randomly selects a single bottle to sample. What is the mean and standard deviation?

B) What is the probability that the psi of the single bottle is 1.3 psi or more below the process mean?

C) Suppose the bottler randomly selected 40 bottles from the last 10,000 produced. What is the mean and standard deviation of the sampling distribution?

D) What is the probability that the sample mean of the 40 bottles is 1.3 psi or more below the process mean?

E) From part c, in order to reduce the standard deviation 50% (half) , how large would the sample size need to be.

4. A study of 10,000 males who smoke at least two packs of cigarettes daily shows that the mean life span is 65.3 years while the standard deviation is 3.4 years. If a sample of 40 is selected, find the probability that the mean life span of the sample is less than the retirement age of 65 years.

5. The U.S. Department of Transportation keeps monthly reports on airline performance. Among the data which they collect is the percentage of flights that arrive on time. For November 2006, 78% of the flights by the 12 major airlines arrive on time. For that period, what is the probability that in a random sample of 100 flights:

A) More than 63% arrive on time?

B) No more than 30% arrive on time.

6. A battery is designed to last for 25 hours of operation during normal use. The batteries are produced in batches of 2000 and 50 batteries from each batch are tested. If the mean life of the sample is less than 24 hours, the entire batch is rejected. Assuming that $\mu = 25$ and $\sigma = 3$ hours for any randomly selected battery, find the probability that a batch will be rejected.

7. A sampling distribution for means has a mean of 18 and a standard deviation of 3.2 when the sample size is n = 50. If I need to reduce the standard deviation to ¼ of that, how large a sample would I need?

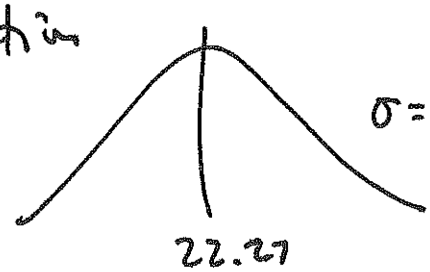
8. A manufacturing process is designed to produce bolts with a 0.5 inch diameter.. Once a day, a random sample of 36 bolts is selected and the diameter recorded. If the resulting sample mean is less than 0.49 inches or greater than 0.51 inches, the process is shut down for adjustment. The standard deviation for the diameter of a single bolt is 0.02 inches. If these bolt diameters are normally distributed, what is the probability that the sample mean falls outside the 0.49 to 0.51 inch interval?

9. Just before a referendum on a school budget, a local newspaper polls 400 voters in an attempt to predict whether the budget will pass. Suppose that the budget actually has the support of 52% of the voters (like magic we know this…). What's the probability that the newspaper's sample will lead them to predict defeat?

10. When a truckload of apples arrives at a packing plant, a random sample of 150 is selected and examined for bruises, discolorations, and other defects. The whole truckload will be rejected if more than 5% of the sample is unsatisfactory. Suppose that in fact 8% of the apples on the truck do not meet the desired standard. What's the probability that the shipment will be accepted anyway?

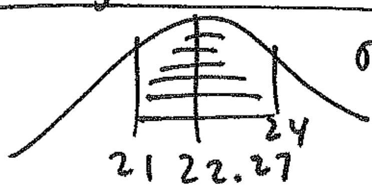
Ex. 1

population



$\sigma = 2.20$

A) a single taxi fare



$\sigma = 2.20$

$$z_1 = \frac{x-\mu}{\sigma} = \frac{24-22.27}{2.20} = 0.79$$

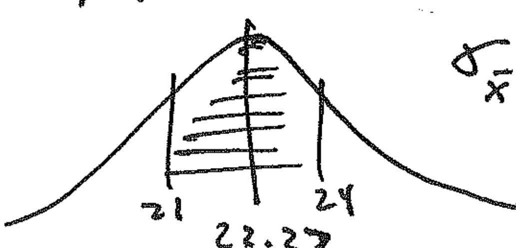
$$z_2 = \frac{x-\mu}{\sigma} = \frac{21-22.27}{2.20} = -0.58$$

$$P(-0.58 < z < 0.79) = .7852 - .2810 = \boxed{.5042}$$

mean of a sample of 10 taxi fares

Check conditions:
random sample
population is normal

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{2.20}{\sqrt{10}} = .6957$$



much more likely that the mean of the sample is between 21 and 24 because sample means have less spread away from the mean than individual values

$$z_1 = \frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}} = \frac{24-22.27}{.6957} = 2.49$$

$$z_2 = \frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}} = \frac{21-22.27}{.6957} = -1.83$$

$$P(-1.83 < z < 2.49) = .9936 - .0336 = \boxed{.96}$$

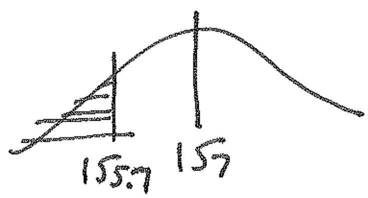
SAMPLING DISTRIBUTIONS WORKSHEET - SOLUTIONS

(1) (B) A single taxi fare because individual fares have more variation than sample means

(2) DID IN CLASS

(3) (A) Since it is a single bottle (individual value) it is the same mean and standard deviation as the population $N(157, 3)$

(B) No conditions to check because it is a single bottle and we were already told this was a normal distribution



$\sigma = 3$

$$z = \frac{x-\mu}{\sigma} = \frac{155.7-157}{3} = -.43$$

$$P(z < -0.43) = .3336$$

(C) $\mu = 157$

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{3}{\sqrt{40}} = .4743$$

(D) random sample

population is normal

$$\sigma_{\bar{x}} = \frac{3}{\sqrt{40}} \qquad z = \frac{\bar{x}-\mu}{\sigma_{\bar{x}}} = \frac{155.7-157}{\frac{3}{\sqrt{40}}} = -2.74$$

$$P(z < -2.74) = .0031$$

(E)
$$\frac{3}{\sqrt{n}} = \frac{1}{2}\left(\frac{3}{\sqrt{40}}\right)$$

$$\frac{3}{\sqrt{n}} = \frac{3}{2\sqrt{40}} \quad \text{cross multiply}$$

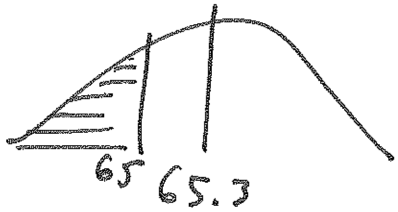
$$\frac{\cancel{3}\sqrt{n}}{\cancel{3}} = \frac{6\sqrt{40}}{3}$$

$\sqrt{n} = 2\sqrt{40}$ square both sides

$n = 160 \Longrightarrow$ when you Quadruple sample size it cuts st. dev. in half

(4) Assume random sample

$n = 40 \geq 30$ so sampling distribution is approx. normal according to CLT



$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{3.4}{\sqrt{40}} = .5376$$

$$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{65 - 65.3}{\left(\frac{3.4}{\sqrt{40}}\right)} = -0.56$$

$$P(z < -0.56) = .2877$$

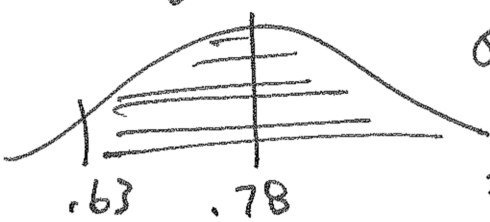
(5) random sample

population of all flights in Nov 2006 $\geq 10(100)$

$np = 100(.78) = 78 \geq 10$

$nq = 100(.22) = 22 \geq 10$

(A)

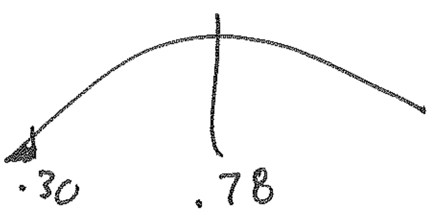


$$\sigma_{\hat{p}} = \sqrt{\frac{pq}{n}} = \sqrt{\frac{.78(.22)}{100}} = .0414$$

$$z = \frac{\hat{p} - p}{\sigma_{\hat{p}}} = \frac{.63 - .78}{.0414} = -3.62$$

$$P(z > -3.62) = .9999 \text{ (from calculator)}$$

(B)

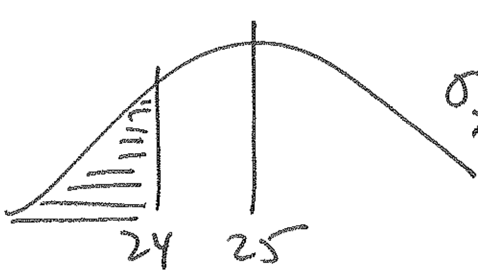


$$z = \frac{.30 - .78}{.0414} = -11.59$$

$$P(z < -11.59) = \text{near } 0$$

(6) Assume random sample of batteries

$n = 50 \geq 30$ so sampling distribution is approx. normal according to CLT



$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{3}{\sqrt{50}} = .4243$$

$$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{24 - 25}{.4243} = -2.36$$

$$P(z < -2.36) = .0091$$

(7) $\sigma_{\bar{x}} = 3.2$ when $n = 50$

$$\frac{\sigma}{\sqrt{50}} = 3.2 \implies \sigma = 3.2\sqrt{50}$$

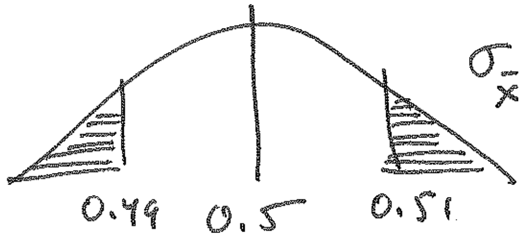
If we cut this st dev to 1/4, then we have

$$\frac{3.2\sqrt{50}}{\sqrt{n}} = 0.8$$ multiply both sides by $\sqrt{n}$ and divide by 0.8 to get

$\sqrt{n} = 28.2843$ Then square both sides

$n = 800$

(8) random sample
population is normally distributed



$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{0.02}{\sqrt{36}} = .0033$$

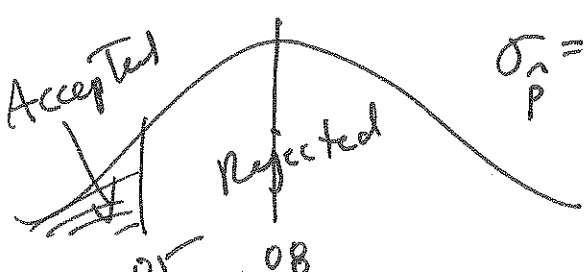
$$z = \frac{\bar{x} - \mu}{\sigma_{\bar{x}}} = \frac{0.49 - 0.5}{.0033} = -3.03$$

$$P(z < -3.03) = .0012$$

Double this due to the symmetry of shaded regions $.0012(2) = .0024$

(9) DID IN CLASS

(10) random sample of apples
population of all apples $\geq 10(150)$
$np = 150(.08) = 12 \geq 10$
$nq = 150(.92) = 138 \geq 10$



$$\sigma_{\hat{p}} = \sqrt{\frac{pq}{n}} = \sqrt{\frac{.08(.92)}{150}} = .0222$$

$$z = \frac{\hat{p} - p}{\sigma_{\hat{p}}} = \frac{.05 - .08}{.0222} = -1.35$$

$$P(z < -1.35) = .0885$$