

12-9

Solving Rational Equations (Pages 690–695)

A **rational equation** is an equation that contains rational expressions. To solve a rational equation, multiply each side of the equation by the LCD of the rational expressions in the equation. Doing so can yield results that are not solutions to the original equation, called **extraneous solutions** or “false” solutions. To eliminate extraneous solutions, be sure no solution is an excluded value of the original equation.

Example

Solve $\frac{a}{a+1} + \frac{3a+4}{a+1} = 3$.

$$\frac{a}{a+1} + \frac{3a+4}{a+1} = 3$$

$$(a+1)\left(\frac{a}{a+1} + \frac{3a+4}{a+1}\right) = (a+1)3 \quad \text{Multiply each side by the LCD, } a+1.$$

$$(a+1)\frac{a}{a+1} + (a+1)\frac{3a+4}{a+1} = (a+1)3 \quad \text{Use the Distributive Property.}$$

$$a + 3a + 4 = 3a + 3 \quad \text{Multiply.}$$

$$4a + 4 = 3a + 3 \quad \text{Add.}$$

$$a + 4 = 3 \quad \text{Subtract } 3a \text{ from each side.}$$

$$a = -1 \quad \text{Subtract 4 from each side.}$$

Since -1 is an excluded value of the original equation, -1 is an extraneous solution. Thus, this equation has no solution.

Practice

Solve each equation.

1. $\frac{2}{3y} + \frac{4}{y} = \frac{1}{3}$

2. $n - 4 = \frac{5}{n}$

3. $\frac{-3}{x} = 7 + 2x$

4. $\frac{1}{t} = \frac{3}{t-6}$

5. $\frac{x-2}{x} + (x+7) = \frac{-9}{x}$

6. $2x = \frac{4x}{x-2}$

7. $\frac{k+8}{k} - \frac{k-4}{k} = 3$

8. $\frac{a+1}{a} = \frac{a+1}{a-4}$

9. $\frac{n-3}{n-1} + \frac{2n}{n-1} = 2$

10. $\frac{w+5}{w+6} + \frac{w}{4} = \frac{1}{4}$

11. $\frac{x}{x+2} = \frac{1}{x}$

12. $\frac{n-1}{n} = \frac{n+1}{n+3}$

13. $\frac{x}{8} + \frac{2}{x} = \frac{x}{4}$

14. $\frac{y+3}{y+2} = 1 - \frac{y+1}{y+2}$

15. $\frac{c+4}{c-2} - 3 = \frac{c}{4}$

16. **Standardized Test Practice** Solve $\frac{x}{3} - \frac{1}{x} = \frac{2}{x}$.

A $x = -3$

B $x = 3$

C $x = -3, 3$

D no solution

Answers: 1. 14 2. -1, 5 3. -3, - $\frac{2}{1}$ 4. -3 5. -7, -1 6. 0, 4 7. 4 8. -1 9. no solution 10. -7, -2 11. -1, 2 12. 3 13. -4, 4 14. no solution 15. -10, 4 16. C